Computer Graphics

Computer and video game programmers use transformations to create patterns and animations. How can you use coordinates to describe the transformation shown?

The transformation is a translation. You can use coordinate notation to describe a translation. An arrow is used in coordinate notation to signify “goes to.”

Coordinates of original point

Coordinates of image

\[(x, y) \rightarrow (x + a, y + b)\]

The number \(a\) tells how many units to shift the figure to the left or right.

The number \(b\) tells how many units to shift the figure up or down.

**EXAMPLE 1 Describing a Translation**

You can use coordinate notation to describe the translation shown above. Each point on the original figure is moved 4 units to the left and 3 units down.

**ANSWER** In coordinate notation you write this translation as:

\[(x, y) \rightarrow (x + (-4), y + (-3))\] or \[(x, y) \rightarrow (x - 4, y - 3)\]

**Your turn now** Describe the translation using coordinate notation.

1. A figure is moved 5 units to the right and 4 units up.
2. A figure is moved 7 units to the left and 2 units up.
EXAMPLE 2 Translating a Figure

Draw quadrilateral $JKLM$ with vertices $J(-5, 3)$, $K(-4, 5)$, $L(-3, 3)$, and $M(-4, 1)$. Then find the coordinates of the vertices of the image after the translation $(x, y) \rightarrow (x + 6, y - 2)$, and draw the image.

For each vertex of the original figure, add 6 to the $x$-coordinate and subtract 2 from the $y$-coordinate.

Original Image

$J(-5, 3) \rightarrow J'(1, 1)$
$K(-4, 5) \rightarrow K'(2, 3)$
$L(-3, 3) \rightarrow L'(3, 1)$
$M(-4, 1) \rightarrow M'(2, -1)$

Each point on the original figure is translated 6 units to the right and 2 units down. The graph shows both figures.

Your turn now Complete the following exercise.

3. Draw $\triangle ABC$ with vertices $A(-4, 0)$, $B(0, -4)$, and $C(0, 0)$. Then find the coordinates of the vertices of the image after the translation $(x, y) \rightarrow (x + 4, y + 6)$, and draw the image.

Reflections You can also use coordinate notation to describe reflections.

Reflection in the $x$-axis:
Multiply $y$-coordinate by $-1$.
$(x, y) \rightarrow (x, -1 \cdot y)$ or $(x, y) \rightarrow (x, -y)$

Reflection in the $y$-axis:
Multiply $x$-coordinate by $-1$.
$(x, y) \rightarrow (-1 \cdot x, y)$ or $(x, y) \rightarrow (-x, y)$
Exercises

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Chapter 10    Geometric Figures

1. Vocabulary
Copy and complete: The figure that results from performing a transformation on an original figure is called the _image_.

Describe the translation using coordinate notation.

2. (x, y) → (x + 2, y + 1)
3. (x, y) → (x - 1, y - 3)

Draw ΔGHJ with vertices G(−4, 1), H(−2, 4), and J(0, 1). Then find the coordinates of the vertices of the image after the specified transformation, and draw the image.

4. (x, y) → (x - 4, y)
5. (x, y) → (x, y + 2)
6. Reflect ΔGHJ in the x-axis.
7. Reflect ΔGHJ in the y-axis.

EXAMPLE 3 Reflecting a Figure

Draw parallelogram ABCD with vertices A(−3, 3), B(2, 3), C(4, 1), and D(−1, 1). Then find the coordinates of the vertices of the image after a reflection in the x-axis, and draw the image.

For each vertex of the original figure, multiply the y-coordinate by −1.

Original    Image
A(−3, 3) → A'(−3, −3)
B(2, 3) → B'(2, −3)
C(4, 1) → C'(4, −1)
D(−1, 1) → D'(−1, −1)

The graph shows both figures.
Describe the transformation using coordinate notation.

8. 

9. 

Draw the triangle with the given vertices. Then find the coordinates of the vertices of the image after the specified translation, and draw the image.

10. \( P(1, 1), Q(3, 5), R(5, 4); (x, y) \rightarrow (x - 2, y - 4) \)

11. \( F(-2, 3), G(3, 3), H(3, -1); (x, y) \rightarrow (x - 3, y - 6) \)

12. \( L(-6, 0), M(-6, -4), N(-3, -4); (x, y) \rightarrow (x, y + 5) \)

Draw rectangle \( FGHJ \) with vertices \( F(-2, 3), G(3, 3), H(3, -1), \) and \( J(-2, -1) \). Then find the coordinates of the vertices of the image after the specified transformation, and draw the image.

13. \( (x, y) \rightarrow (x + 3, y + 6) \)

14. \( (x, y) \rightarrow (x - 7, y) \)

15. Reflect \( FGHJ \) in the \( x \)-axis.

16. Reflect \( FGHJ \) in the \( y \)-axis.

17. **Motion**

   Transformations can be used to show motion. Use coordinate notation to describe the transformation of the drummer from one picture to the next picture.

18. **Explain**

   Draw a rectangle with vertices \( K(0, 0), L(-3, 0), M(-3, 4), \) and \( N(0, 4) \). Find the length, width, and area of the rectangle. Then translate the rectangle 2 units to the right. Find the length, width, and area of the image. Explain your results.

19. **Critical Thinking**

   Draw one triangle with vertices \( (2, 1), (3, 4), \) and \( (4, 1) \) and a second triangle with vertices \( (-4, 1), (-3, 4), \) and \( (-2, 1) \). Describe two different transformations that could move the first triangle onto the second.
Rotations In Exercises 20–23, the vertices of a triangle are given. Find the coordinates of the vertices of the image after a rotation of $90^\circ$ clockwise about the origin. The coordinate notation $(x, y) \to (y, -x)$ describes such a rotation.

20. $F(0, 2), G(-3, 1), H(-1, 1)$
21. $L(2, 2), M(4, -1), N(2, -2)$
22. $R(-3, 3), S(-3, 0), T(-1, 0)$
23. $W(0, 0), X(5, -3), Y(3, 4)$

24. Critical Thinking The vertices of quadrilateral $ABCD$ are $A(-3, 4), B(2, 4), C(3, 2),$ and $D(-4, -1)$. After a translation, the coordinates of $A'$ are $(-5, 1)$. Describe the translation using coordinate notation. Then find the coordinates of $B', C',$ and $D'$.

25. Challenge Line segment $EF$ has endpoints $E(4, 3)$ and $F(4, -3)$. Its image after a $180^\circ$ rotation has endpoints $E'(-4, -3)$ and $F'(-4, 3)$. Describe a $180^\circ$ rotation using coordinate notation.

Mixed Review

Solve the equation. Check your solution. (Lesson 7.5)

26. $3x + 5 = 23$  
27. $8 - 4x = -20$  
28. $35 = 3 - 16x$

29. Choose a Strategy Use a strategy from the list to solve the following problem. Explain your choice of strategy.

You paid three times as much for lunch as you paid for a snack. Altogether, you spent $5. How much did you pay for the snack? for lunch?

Basic Skills Copy and complete the statement.

30. $0.53 \text{ kg} = \text{?} \text{ g}$  
31. $8.9 \text{ m} = \text{?} \text{ cm}$  
32. $62 \text{ mL} = \text{?} \text{ L}$

Test-Taking Practice

33. Extended Response Describe a combination of two transformations that would move figure A to figure B. Then describe a combination of two transformations that would move figure B to figure A.