### Permutations and Combinations

#### Activity
You can investigate numbers of arrangements.

1. Line up 3 chairs and choose 3 students.

2. How many ways can 1 of the 3 students be chosen to sit in the first chair? Have one of the students sit in the first chair.

3. Repeat Step 2 for the second and third chairs.

4. Apply the counting principle to your answers from Steps 2 and 3 to find the number of arrangements of 3 students in 3 chairs.

In the activity, you found the number of permutations of 3 students. A permutation is an arrangement of a group of objects in a particular order. For example, the 6 permutations of 3 letters in the word CAT are shown below.

<table>
<thead>
<tr>
<th>CAT</th>
<th>ACT</th>
<th>TCA</th>
</tr>
</thead>
<tbody>
<tr>
<td>CTA</td>
<td>ATC</td>
<td>TAC</td>
</tr>
</tbody>
</table>

#### Example 1
**Counting Permutations**

**Amusement Parks** You and 3 of your friends go to an amusement park. How many ways can you stand in line to buy tickets for the rides?

**Solution**
Use the counting principle.

\[
\text{Choices for first in line} \times \text{Choices for second in line} \times \text{Choices for third in line} \times \text{Choices for fourth in line} = \text{Ways to stand in line}
\]

\[
4 \times 3 \times 2 \times 1 = 24
\]

**Answer** There are 24 ways that you and your 3 friends can stand in line.
Lesson 13.4    Permutations and Combinations

Example 2  Counting Permutations

Stamp Competition  There were 52 entries in the 2002–2003 Federal Junior Duck Stamp contest. In how many ways could the first, second, and third places be awarded?

Solution

Choices for first place × Choices for second place × Choices for third place = Ways to award first, second, and third

52 × 51 × 50 = 132,600

ANSWER  There were 132,600 ways to award first, second, and third places.

Example 3  Listing Combinations

School Electives  You need to choose 2 different electives from the following 4 electives: Spanish (S), consumer and family studies (C), industrial technology (I), and art (A). How many different choices do you have, if the order in which you choose the classes does not matter?

Solution

Start by listing all of the permutations of 2 electives. Because the order in which you choose the electives does not matter, cross out one of any pair of permutations that lists the same two electives.

SC  SI  SA
CS  CI  CA
IS  IC  IA
AS  AC  AI

Because IA and AI list the same electives, cross one of them out.

ANSWER  You have 6 different choices for choosing 2 electives.

Your turn now  Find the number of permutations.

1. In how many ways can you arrange the letters in the word COMPUTER?
2. There are 8 volleyball teams in a tournament. In how many ways can teams place first, second, third, and fourth?

Combinations  In a permutation, the order of the objects is important. A combination is a grouping of objects in which the order is not important.

Help with Solving

In Example 3, you can use a tree diagram to find the permutations of 2 electives. Then cross out one of any pair of permutations that lists the same two electives.
Relating Permutations and Combinations  In Example 3, another way to find the number of combinations is to divide the number of permutations when choosing 2 electives from 4 by the number of permutations when arranging 2 electives, as shown below.

\[
\frac{\text{Permutations when choosing 2 objects from 4}}{\text{Permutations when arranging 2 objects}} = \frac{4 \times 3}{2 \times 1} = 6 \text{ combinations}
\]

This method is useful when there are too many arrangements to list.

Example 4  Relating Combinations and Permutations

Sunglasses  You win a door prize at the grand opening of a department store. For your prize, you get to choose 5 different pairs of sunglasses from the 20 styles that the store carries. How many choices do you have?

Solution  
Because the order in which the sunglasses are chosen does not matter, you need to find the number of combinations.

1. Find the number of permutations when choosing 5 pairs from 20 styles.
   \[20 \times 19 \times 18 \times 17 \times 16 = 1,860,480\]

2. Find the number of permutations when arranging 5 objects.
   \[5 \times 4 \times 3 \times 2 \times 1 = 120\]

3. Divide the number of permutations when choosing 5 pairs from 20 styles by the number of permutations when arranging 5 objects.
   \[\frac{1,860,480}{120} = 15,504\]

ANSWER  There are 15,504 ways for you to choose the sunglasses.

Your turn now

Find the number of combinations.

3. You want to buy 5 CDs at a music store. If you have enough money for only 2 CDs, how many choices do you have?

4. As part of a summer reading program, you need to read 4 books. Your school gives you a list of 15 books from which to choose. How many ways can you choose the 4 books if the order in which you choose them is not important?
Getting Ready to Practice

Vocabulary  Copy and complete the statement.

1. A(n) ? is a grouping of objects in which the order is not important.
2. A(n) ? is an arrangement of a group of objects in a particular order.
3. Relay Race  There are 12 members of a track team who want to run one of the legs in a 4 person relay race. Choose the calculation that you can use to find the number of ways that runners can be chosen for each of the legs of the relay race.

   A.  $12 \times 11 \times 10 \times 9$

   B.  $4 \times 3 \times 2 \times 1$

   C.  $\frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2 \times 1}$

4. Guided Problem Solving  You need to choose 3 different colors of balloons to be used as decorations at a school graduation. The balloons are available in 24 colors. How many ways can you choose 3 different colors of balloons?

   (1) Decide whether the situation describes a permutation or a combination.

   (2) Find the number of permutations or combinations.

Practice and Problem Solving

Simplify the expression.

5.  $5 \times 4 \times 3 \times 2 \times 1$

6.  $\frac{11 \times 10 \times 9}{3 \times 2 \times 1}$

7.  $\frac{20 \times 19 \times 18 \times 17}{4 \times 3 \times 2 \times 1}$

Find the number of permutations.

8. Ways to arrange the letters in the word GUITAR

9. Ways to arrange 7 DVDs on a shelf

10. Ways to choose a president, vice-president, treasurer, and secretary from the 18 members of a club

11. Ways to choose 3 different posters from 27 posters

12. Critical Thinking  Decide whether the list below contains all of the permutations of the 3 symbols. If the list is incomplete, determine which permutations are missing.
Find the number of combinations.

13. Ways to choose 3 different kinds of apples from the following kinds: Red Delicious, Granny Smith, Empire, McIntosh, and Fuji

14. Ways to choose 4 different colors from the following colors: red, blue, purple, yellow, green, and orange

15. Ways to choose 8 students to be extras in a play from 14 students

16. Ways to choose 4 different fish from 26 kinds of fish

Cards In Exercises 17 and 18, use the following information. As part of a magic trick, you are asked to choose a number of cards from a deck of 52 cards.

17. How many ways can you choose 4 cards from the deck, if the order in which the cards are chosen is not important?

18. How many ways can you choose 6 cards from the deck, if the order in which the cards are chosen is important?

Tell whether the situation describes a permutation or a combination. Then answer the question.

19. How many ways can a disc jockey choose 4 different songs from 10?

20. How many ways can you choose 3 different pizza toppings from 15?

21. How many ways can a judge award first, second, and third places at a science fair with 23 entries?

22. How many ways can the coach arrange the batting order of the 9 starting players of a baseball team?

23. Writing Describe a real-world situation that involves a permutation and a real-world situation that involves a combination.

In Exercises 24 and 25, answer the questions about the word ALASKA.

24. Why can't you use the expression $6 \times 5 \times 4 \times 3 \times 2 \times 1$ to find the number of different arrangements of the letters in the word?

25. How many different ways can the letters in the word be arranged?

26. Computers A group of 40 computers contains exactly 2 defective computers. Suppose 2 computers are randomly selected from the group. What is the probability that the 2 computers are defective? Write the probability as a decimal rounded to the nearest thousandth.

27. Challenge How many different 3 digit numbers can you make using the digits 1, 4, 5, 6, 8, and 9, if no digit appears more than once in the number?
Mixed Review

You randomly choose a marble from the marbles below. Find the probability of choosing a marble of the given color. Write the probability as a fraction, a decimal, and a percent. (Lesson 13.1)


32. At a restaurant, you can have your eggs cooked 4 different ways, and you can choose from 3 kinds of bread for your toast. How many eggs and toast combinations are possible? (Lesson 13.3)

Basic Skills

Find the sum or difference.

33. \( \frac{1}{8} + \frac{3}{8} \)  34. \( \frac{2}{7} + \frac{4}{9} \)  35. \( \frac{8}{13} - \frac{6}{13} \)  36. \( \frac{1}{5} - \frac{1}{6} \)

Test-Taking Practice

37. Multiple Choice  There are 8 students participating in a car wash. How many ways can 2 of the students be chosen to hold signs advertising the car wash?

A. 8  B. 16  C. 28  D. 56

38. Short Response  A bag contains 1 green marble, 1 blue marble, 1 red marble, and 1 white marble. How many ways can 3 marbles be randomly chosen from the bag, if the order in which the marbles are chosen is important?

Creating Quadrilaterals

Ten pegs are arranged in a circle on a board. A rubber band is stretched to create a quadrilateral as shown. How many quadrilaterals can be created? (If two quadrilaterals have the same shape but are made using different pegs, then the quadrilaterals are different.)