Make this Foldable to help you organize information about the material in this chapter. Begin with four sheets of 8½ by 11” lined paper.

1. **Stack** sheets of paper with edges ¾ inch apart.
2. **Fold** up bottom edges. All tabs should be the same size.
3. **Staple** along the fold.
4. **Label** the tabs with topics from the chapter.

**Reading and Writing** As you read and study the chapter, use each page to write notes and examples for each lesson.
Problem-Solving Workshop

Project
As a new video club member, you can choose seven movies. Each movie costs 1¢ plus $1.69 for shipping and handling. Within three years, you must order at least five more movies, each at the regular club price, plus the same shipping fee.

Suppose you buy a total of 12 movies. What is the lowest possible average cost per movie? the highest possible average cost per movie?

<table>
<thead>
<tr>
<th>Type of Movie</th>
<th>Regular Club Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Children’s</td>
<td>$12.99</td>
</tr>
<tr>
<td>New Release</td>
<td>$24.99</td>
</tr>
<tr>
<td>All-Time Favorite</td>
<td>$16.99</td>
</tr>
<tr>
<td>Classic</td>
<td>$8.99</td>
</tr>
</tbody>
</table>

Working on the Project
Work with a partner and choose a strategy to help analyze and solve the problem. Here are some questions to help you get started.

• How much do you pay for the first shipment of seven 1¢ movies?
• What are the least and greatest amounts you can spend on the five required regular-priced movies?

Technology Tools
• Use a spreadsheet to calculate the average cost of the movies.
• Use word processing software to write your newspaper article.

Research
For more information about CD clubs, visit: www.algconcepts.com

Presenting the Project
Write an article for the school newspaper discussing the advantages and disadvantages of joining a video, book, or CD club.

• Research prices at retail or online stores. What would you pay for the same number of videos, books, or CDs required by the club?
• Show how the average cost per video, book, or CD changes as you buy more at the regular club price.
Writing Expressions and Equations

Suppose a candy bar costs 45 cents. Then 45 \times 2 is the cost of 2 candy bars, 45 \times 3 is the cost of 3 candy bars, and so on. Generally, the cost of any number of candy bars is 45 \text{ cents times the number of bars}. We can represent this situation with an \textbf{algebraic expression}.

\[
45 \text{ cents times the number of bars} = 45 \times n
\]

The letter \(n\) stands for an unknown number, in this case, candy bars. The unknown \(n\) is called a \textbf{variable} because its value \textit{varies}. An algebraic expression contains at least one variable and at least one mathematical operation, as shown in the examples below.

\[
h \div 3 \quad 5n + 1 \quad \frac{r}{t} - 1 \quad xy \quad 4 \times a
\]

A \textbf{numerical expression} contains only numbers and mathematical operations. For example, \(6 + 2 \div 1\) is a numerical expression.

In an expression involving multiplication, the quantities being multiplied are called \textbf{factors}, and the result is the \textbf{product}.

\[
4 \times 5 \times 8 = 160
\]

To write a multiplication expression such as \(4 \times a\), a raised dot or parentheses can be used. A fraction bar can be used to represent division.

\[
\frac{4 \cdot a}{4(a)} \quad \frac{4(a)}{(4)(a)} \quad \frac{t}{2}
\]

The result of a division expression is called a \textbf{quotient}.

To solve verbal problems in mathematics, you may have to translate words into algebraic expressions. The chart below shows some of the words and phrases used to indicate mathematical operations.

<table>
<thead>
<tr>
<th>Addition</th>
<th>Subtraction</th>
<th>Multiplication</th>
<th>Division</th>
</tr>
</thead>
<tbody>
<tr>
<td>plus</td>
<td>minus</td>
<td>times</td>
<td>divided by</td>
</tr>
<tr>
<td>the sum of</td>
<td>the difference of</td>
<td>the product of</td>
<td>the quotient of</td>
</tr>
<tr>
<td>increased by</td>
<td>decreased by</td>
<td>multiplied by</td>
<td>the ratio of</td>
</tr>
<tr>
<td>more than</td>
<td>less than</td>
<td>at</td>
<td>per</td>
</tr>
<tr>
<td>added to</td>
<td>fewer than</td>
<td>of</td>
<td></td>
</tr>
<tr>
<td>the total of</td>
<td>subtracted from</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[4 \text{ Chapter 1 The Language of Algebra}\]
Write an algebraic expression for each verbal expression.

1. the sum of \( m \) and 18
   \[ m + 18 \]

2. \( g \) divided by \( y \)
   \[ g \div y \text{ or } \frac{g}{y} \]

Your Turn

a. 26 decreased by \( w \)
   \[ 26 - w \]

b. 4 more than 8 times \( k \)
   \[ 8k + 4 \]

Some kangaroos can travel 30 feet in a single leap.

3. Write a numerical expression to represent the distance a kangaroo can travel if it leaps 4 times.
   \[ 30 \times 4 \text{ or } 30(4) \]

4. Write an algebraic expression to represent the distance a kangaroo can travel if it leaps \( x \) times.
   \[ 30 \times x \text{ or } 30x \]

You can also translate algebraic expressions into verbal expressions.

Examples

Write a verbal expression for each algebraic expression.

5. \( 32 - b \)
   32 less \( b \)
   \( b \) less than 32
   the difference of 32 and \( b \)
   \( b \) subtracted from 32
   32 decreased by \( b \)

6. \( (y + 4) + 9 \)
   \( y \) divided by \( 4 \), plus 9
   the quotient of \( y \) and 4, increased by 9
   9 added to the ratio of \( y \) and 4

Your Turn

c. 15\(v\)

d. \( r - \frac{t}{d} \)

An equation is a mathematical sentence that contains an equals sign (\( = \)). Some words used to indicate the equals sign are in the chart at the right. An equation may contain numbers, variables, or algebraic expressions.

Examples

Write an equation for each sentence.

7. Three times \( g \) equals 21.
   \[ 3g = 21 \]

8. Five more than twice \( n \) is 15.
   \[ 2n + 5 = 15 \]

Your Turn

e. A number \( k \) divided by 4 is equal to 18.
   \[ \frac{k}{4} = 18 \]
Check for Understanding

Communicating Mathematics

1. Write three examples of numerical expressions and three examples of algebraic expressions.
2. Write three examples of equations.
3. Write about a real-life application that can be expressed using an algebraic expression or an equation.

Guided Practice

Write an algebraic expression for each verbal expression. (Examples 1 & 2)

4. \( t \) more than \( s \)
5. the product of 7 and \( m \)
6. 11 decreased by the quotient of \( x \) and 2

Write a verbal expression for each algebraic expression. (Examples 5 & 6)

7. \( \frac{7}{q} \)
8. \( 3\ell - 9 \)

Write an equation for each sentence. (Examples 7 & 8)

9. A number \( m \) added to 6 equals 17.
10. Ten is the same as four times \( r \) minus 6.

Write a sentence for each equation. (Examples 9 & 10)

11. \( 5 + r = 15 \)
12. \( \frac{4p}{3} = 12 \)

13. Biology The family of great white sharks has \( w \) different species. The blue shark family has nine times \( w \) plus three different species. Write an algebraic expression to represent the number of species in the blue shark family. (Example 4)
Write an algebraic expression for each verbal expression.

14. twelve less than $y$

15. the product of $r$ and $s$

16. the quotient of $t$ and 5

17. three more than five times $a$

18. $p$ plus the quotient of 9 and 5

19. the difference of 1 and $n$

20. $f$ divided by $y$

21. ten plus the product of $h$ and 1

22. seven less than the quotient of $j$ and $p$

Write a verbal expression for each algebraic expression.

23. $9x$

24. $11 + b$

25. $6 - y$

26. $2m + 1$

27. $\frac{3}{r} - 8$

28. $16 - rt$

Write an equation for each sentence.

29. Three plus $w$ equals 15.

30. Five times $r$ equals 7.

31. Two is equal to seven divided by $x$.

32. Five less than the product of two and $g$ equals nine.

33. Three minus the product of five and $y$ is the same as two times $z$.

34. The quotient of 19 and $j$ is equal to the total of $a$ and $b$ and $c$.

Write a sentence for each equation.

35. $3r = 18$

36. $g + 7 = 3$

37. $h = 10 - i$

38. $6v - 2 = 8$

39. $\frac{1}{4} = 16$

40. $10z + 7 = \frac{6}{r}$

41. Choose a variable and write an equation for Four times a number minus seven equals the sum of 15 and $c$ and two times the number.

42. Biology A smile requires 26 fewer muscles than a frown. Let $f$ represent the number of muscles it takes for a frown and let $s$ represent the number of muscles for a smile. Write an equation to represent the number of muscles a person uses to smile.

43. Communication A long-distance telephone call costs 20¢ for the first minute plus 10¢ for each additional minute.

a. Write an expression for the total cost of a call that lasts 15 minutes.

b. Write an expression for the total cost of a call that lasts $m$ minutes.

44. Critical Thinking The ancient Hindus enjoyed number puzzles like the one below. Source: Mathematical History

If 4 is added to a certain number, the result divided by 2, that result multiplied by 5, and then 6 subtracted from that result, the answer is 29. Can you find the number?

a. Choose a variable and write an algebraic equation to represent the puzzle.

b. Is your answer in part a the only correct way to write the equation? Explain.
Some expressions have more than one operation. The value of the expression depends on the order in which the operations are evaluated. What is the value of \(9 \div 5 + 4\)?

**Method 1**  
\[
9 \div 5 + 4 = 18 + 4 = 22
\]

**Method 2**  
\[
9 \div 5 + 4 = 9 \cdot 9 = 81
\]

Is the answer 22 or 81? The values are different because we multiplied and added in different orders in the two methods. To find the correct value of the expression, follow the **order of operations**.

According to the order of operations, do multiplication and then addition. So, the value of the expression in Method 1 is correct. The value of the expression is 22.

**Examples**

1. Find the value of each expression.

\[
38 - 5 \cdot 6
\]

\[
38 - 5 \cdot 6 = 38 - 30 = 8
\]

2. Evaluate the numerator and the denominator separately.

\[
\frac{4 \times 9}{26 - 8}
\]

\[
\frac{4 \times 9}{26 - 8} = \frac{36}{18} = 2
\]

**Your Turn**

a. \(7 \cdot 4 + 7 \cdot 3\)  
b. \(12 \div 3 - 5 - 4\)  
c. \(\frac{6 + 12}{5(3) - 13}\)

---

**Order of Operations**

1. Find the values of expressions inside grouping symbols, such as parentheses ( ), brackets [ ], and as indicated by fraction bars.
2. Do all multiplications and/or divisions from left to right.
3. Do all additions and/or subtractions from left to right.
The order of operations is useful in solving problems in everyday life.

As a 16-year old, Trent Eisenberg ran his own consulting company called *F1 Computer*. Suppose he charged a flat fee of $50, plus $25 per hour. One day he worked 2 hours for one customer and the next day he worked 3 hours for the same customer. Find the value of the expression \( 50 + 25(2 + 3) \) to find the total amount of money he earned.

Source: Scholastic Math

\[
50 + 25(2 + 3) = 50 + 25(5) \\
= 50 + 125 \\
= 175
\]

Trent earned $175.

In algebra, statements that are true for any number are called properties. Four properties of equality are listed in the table below.

<table>
<thead>
<tr>
<th>Property of Equality</th>
<th>Symbols</th>
<th>Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Substitution</td>
<td>If (a = b), then (a) may be replaced by (b).</td>
<td>If (9 + 2 = 11), then (9 + 2) may be replaced by (11).</td>
</tr>
<tr>
<td>Reflexive</td>
<td>(a = a)</td>
<td>(21 = 21)</td>
</tr>
<tr>
<td>Symmetric</td>
<td>If (a = b), then (b = a).</td>
<td>If (10 = 4 + 6), then (4 + 6 = 10).</td>
</tr>
<tr>
<td>Transitive</td>
<td>If (a = b) and (b = c), then (a = c).</td>
<td>If (3 + 5 = 8) and (8 = 2(4)), then (3 + 5 = 2(4)).</td>
</tr>
</tbody>
</table>

Name the property of equality shown by each statement.

4. If \(9 + 3 = 12\), then \(12 = 9 + 3\).
   Symmetric Property of Equality

5. If \(z = 8\), then \(z + 4 = 8 + 4\).
   Substitution Property of Equality  \(z\) is replaced by 8.

**Your Turn**

d. \(7 - c = 7 - c\)
e. If \(10 - 3 = 4 + 3\) and \(4 + 3 = 7\), then \(10 - 3 = 7\).
These properties of numbers may help to find the value of expressions.

<table>
<thead>
<tr>
<th>Property</th>
<th>Words</th>
<th>Symbols</th>
<th>Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Additive Identity</td>
<td>When 0 is added to any number ( a ), the sum is ( a ).</td>
<td>For any number ( a ), ( a + 0 = a ).</td>
<td>( 45 + 0 = 45 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0 + 6 = 6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( 0 = ) the identity.</td>
</tr>
<tr>
<td>Multiplicative Identity</td>
<td>When a number ( a ) is multiplied by 1, the product is ( a ).</td>
<td>For any number ( a ), ( a \cdot 1 = a ).</td>
<td>( 12 \cdot 1 = 12 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1 \cdot 5 = 5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1 = ) the identity.</td>
</tr>
<tr>
<td>Multiplicative Property of Zero</td>
<td>If ( 0 ) is a factor, the product is ( 0 ).</td>
<td>For any number ( a ), ( a \cdot 0 = 0 ).</td>
<td>( 7 \cdot 0 = 0 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0 \cdot 23 = 0</td>
</tr>
</tbody>
</table>

When two or more sets of grouping symbols are used, simplify within the innermost grouping symbols first.

**Example 6**

Find the value of \( 5[3 - (6 \div 2)] + 14 \). Identify the properties used.

\[
5[3 - (6 \div 2)] + 14 = 5[3 - 3] + 14 \quad \text{Substitution Property of Equality} \\
= 5(0) + 14 \quad \text{Substitution Property of Equality} \\
= 0 + 14 \quad \text{Multiplicative Property of Zero} \\
= 14 \quad \text{Additive Identity}
\]

**Your Turn**

f. \( (22 - 15) \div 7 \cdot 9 \)  
g. \( 8 \div 4 \cdot 6(5 - 4) \)

You can also apply the properties of numbers to find the value of an algebraic expression. This is called **evaluating** an expression. Replace the variables with known values and then use the order of operations.

**Examples**

Evaluate each expression if \( a = 9 \) and \( b = 1 \).

**7**

\[
7 + \left( \frac{a}{b} - 9 \right) = 7 + \left( \frac{9}{1} - 9 \right) \quad \text{Replace } a \text{ with } 9 \text{ and } b \text{ with } 1. \\
= 7 + (9 - 9) \quad \text{Substitution Property of Equality} \\
= 7 + 0 \quad \text{Substitution Property of equality} \\
= 7 \quad \text{Additive Identity}
\]

**8**

\[
(a + 4) - 3 \cdot b = (9 + 4) - 3 \cdot 1 \quad \text{Replace } a \text{ with } 9 \text{ and } b \text{ with } 1. \\
= (13) - 3 \cdot 1 \quad \text{Substitution Property of Equality} \\
= 13 - 3 \quad \text{Multiplicative Identity} \\
= 10 \quad \text{Substitution Property of Equality}
\]

**Your Turn**

Evaluate each expression if \( m = 8 \) and \( p = 2 \).

h. \( 6 \cdot p - m + p \)  
i. \( [m + 2(3 + p)] \div 2 \)
Check for Understanding

1. Name two of the three types of grouping symbols discussed in this lesson.
2. Translate the verbal expression six plus twelve divided by three and the sum of six and twelve divided by three into numerical expressions. Use grouping symbols. Evaluate the expressions and explain why they are different.
3. Label a section of your math journal “Toolbox.” Record all properties given in this course, beginning with this lesson.
4. State which operation to perform first.
5. Evaluate each algebraic expression if \( q = 4 \) and \( r = 1 \). (Examples 7 & 8)

Guided Practice

Getting Ready
State which operation to perform first.

Sample: \( 3 + 2 \cdot 4 \)
Solution: Multiply 2 and 4.

Find the value of each expression. (Examples 1–3)

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>4. ( 8 \div 4 \cdot 2 )</td>
<td>5. ( 12 - 6 \cdot 2 )</td>
</tr>
<tr>
<td>6. ( 5(7 + 7) )</td>
<td>7. ( (10 - 4) \div 3 )</td>
</tr>
</tbody>
</table>

Find the value of each expression. Identify the property used in each step. (Example 6)

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>8. ( 7 \cdot 4 + 3 )</td>
<td>9. ( 4(1 + 5) \div 8 )</td>
</tr>
</tbody>
</table>

Name the property of equality shown by each statement. (Examples 4 & 5)

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>11. If ( 5 + 2n = 5 + 3 ) and ( 5 + 3 = 2 \cdot 4 ), then ( 5 + 2n = 2 \cdot 4 ).</td>
<td></td>
</tr>
<tr>
<td>12. If ( \frac{y}{2} = 19 ), then ( 19 = \frac{y}{2} ).</td>
<td></td>
</tr>
</tbody>
</table>

Find the value of each expression. (Examples 7 & 8)

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>13. ( 8(4 - 8 \div 2) )</td>
<td>14. ( 5(2) \cdot (15 \div 15) )</td>
</tr>
</tbody>
</table>

Evaluate each algebraic expression if \( q = 4 \) and \( r = 1 \). (Examples 7 & 8)

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>15. ( 4(q - 2r) )</td>
<td>16. ( \frac{7q}{r + 3} )</td>
</tr>
<tr>
<td>18. Car Rental The cost to rent a car is given by the expression ( 25d + 0.10m ), where ( d ) is the number of days and ( m ) is the number of miles. If Teresa rents the car for five days and drives 300 miles, what is the cost? (Examples 7 &amp; 8)</td>
<td></td>
</tr>
</tbody>
</table>

Exercises

Find the value of each expression.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>19. ( 36 \div 4 + 5 )</td>
<td>20. ( 16 - 4 \cdot 4 )</td>
</tr>
<tr>
<td>22. ( 42 - (24 \div 2) + 10 )</td>
<td>23. ( 42 - 24 \div (2 + 10) )</td>
</tr>
<tr>
<td>25. ( \frac{7(3 + 6)}{3} )</td>
<td>26. ( \frac{4(8 - 2)}{2 \times 2} )</td>
</tr>
</tbody>
</table>
Name the property of equality shown by each statement.

28. If \( x + 3 = 5 \) and \( x = 2 \), then \( 2 + 3 = 5 \).
29. \( 8t - 1 = 8t - 1 \)
30. If \( 6 = 3 + 3 \), then \( 3 + 3 = 6 \).
31. \( \frac{20 - 2}{9} = \frac{18}{9} \)
32. If \( 4 \cdot (7 - 7) = 4 \cdot 0 \) and \( 4 \cdot 0 = 0 \), then \( 4 \cdot (7 - 7) = 0 \).
33. \( a + 1 = 15x \) and \( 15x = 30 \), so \( a + 1 = 30 \).

Find the value of each expression. Identify the property used in each step.

34. \( 7(10 - 1 \cdot 3) \)
35. \( 8(9 - 3 \cdot 2) \)
36. \( 19 - 15 \div 5 \cdot 2 \)
37. \( 10(6 - 5) - (20 \div 2) \)
38. \( \frac{9 \cdot 9 - 1}{3(1 + 2) - 1} \)
39. \( 6(12 - 48 \div 4) + 7 \cdot 1 \)

Evaluate each algebraic expression if \( j = 5 \) and \( s = 2 \).

40. \( 7j - 3s \)
41. \( j(3s + 4) \)
42. \( j + 5s - 7 \)
43. \( \frac{9 \cdot 4 + 5 \cdot s}{7 - j} \)
44. \( \frac{14 + s}{2(j - 1)} \)
45. \( \frac{4js}{s - 1} \)
46. \( 50 \div js + 6 \)
47. \( (3s - j)(5s - j) \)
48. \( [3j - s(4 + s)] \div 3 \)
49. \( 2[16 - (j - s)] \)

50. a. Write an algebraic expression for nine added to the quantity three times the difference of \( a \) and \( b \).
   b. Let \( a = 4 \) and \( b = 1 \). Evaluate the expression in part a.

51. **Real Estate**  The Phams own a $150,000 home in Rochester, New York, and plan to move to San Diego, California. How much will a similar home in San Diego cost? Evaluate the expression \( 150,000 \div a \times b \) for \( a = 79 \) and \( b = 164 \) to find the answer to the nearest dollar.
   **Source:** USA TODAY

52. **Sports**  A person’s handicap in bowling is usually found by subtracting the person’s average \( a \) from 200, multiplying by 2, and dividing by 3.
   a. Write an algebraic expression for a handicap in bowling.
   b. Find a person’s handicap whose average is 170.

53. **Gardening**  Mr. Martin is building a fence around a rectangular garden, as shown at the left. Evaluate the expression \( 2\ell + 2w \), where \( \ell \) represents the length and \( w \) represents the width, to find how much fencing he needs.
54. **Critical Thinking**  The symbol $<$ means “is less than.” Are the following properties of equality true for statements containing this symbol? Give examples to explain.
   a. Reflexive  
   b. Symmetric  
   c. Transitive

**Mixed Review**

**Write a sentence for each equation.**  (*Lesson 1–1*)

55. $x + 8 = 12$  
56. $2y = 16$  
57. $25 \div n = 5$

**Write an equation for each sentence.**  (*Lesson 1–1*)

58. Six more than $g$ is 22.  
59. Three times $c$ equals 27.  
60. Two is the same as the quotient of 8 and $x$.  
61. $b$ increased by 10 and then decreased by 1 is equivalent to 18.

**Time**  How many seconds are there in a day?  (*Lesson 1–1*)
   a. Write an expression to answer this question.  
   b. Evaluate the expression.

**Extended Response**  Lincoln’s Gettysburg Address began “Four score and seven years ago, . . .”  (*Lesson 1–1*)
   a. A score is 20. Write a numerical expression for the phrase.  
   b. Evaluate the expression to find the number of years.

**Multiple Choice**  At the movie theater, the price for an adult ticket $a$ is $1.50 less than two times the price of a student ticket $s$. Choose the algebraic expression that represents the price of an adult ticket in terms of the price of a student ticket.  (*Lesson 1–2*)
   A $1.50 - 2s$  
   B $2(s - 1.50)$  
   C $2s + 2(1.50)$  
   D $2s - 1.50$

---

**Quiz 1 Lessons 1–1 and 1–2**

1. Write an algebraic expression for five less than the product of two and $v$.  (*Lesson 1–1*)

2. Write an equation for the sum of nine and $y$ equals 16.  (*Lesson 1–1*)

**Evaluate each algebraic expression if $j = 5$ and $s = 2$.**  (*Lesson 1–2*)

3. $3(j - s) + 4$  
4. $[(11 - j \div 1) + 8] \cdot s$

5. **Carpentry**  Ana Martinez is putting molding around the ceiling of her family room. The room measures 12 feet by 16 feet. Evaluate the expression $2l + 2w$, where $l$ is the length and $w$ is the width, to find how much molding Ana needs.  (*Lesson 1–2*)
The **Commutative Property of Addition** states that the sum of two numbers does not depend on the order in which they are added. In the example below, adding 35 and 50 in either order does not change the sum.

\[
\begin{align*}
35 + 50 &= 85 \\
50 + 35 &= 85
\end{align*}
\]

This example illustrates the Commutative Property of Addition.

Likewise, the order in which you multiply numbers does not matter.

<table>
<thead>
<tr>
<th>Commutative Property of Multiplication</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Words:</strong> The order in which two numbers are multiplied does not change their product.</td>
</tr>
<tr>
<td><strong>Symbols:</strong> For any numbers (a) and (b), (a \cdot b = b \cdot a).</td>
</tr>
<tr>
<td><strong>Numbers:</strong> (3 \cdot 10 = 10 \cdot 3)</td>
</tr>
</tbody>
</table>

Some expressions are easier to evaluate if you group or **associate** certain numbers. Look at the expression below.

\[
16 + 7 + 3 = 16 + (7 + 3) \quad \text{Group 7 and 3.}
\]
\[
= 16 + 10 \quad \text{Add 7 and 3.}
\]
\[
= 26 \quad \text{Add 16 and 10.}
\]

This is an application of the **Associative Property of Addition**.
The Associative Property also holds true for multiplication.

<table>
<thead>
<tr>
<th>Associative Property of Multiplication</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Words:</strong> The way in which three numbers are grouped when they are multiplied does not change their product.</td>
</tr>
<tr>
<td><strong>Symbols:</strong> For any numbers ( a, b, ) and ( c, (a \cdot b) \cdot c = a \cdot (b \cdot c). )</td>
</tr>
<tr>
<td><strong>Numbers:</strong> ((9 \cdot 4) \cdot 25 = 9 \cdot (4 \cdot 25))</td>
</tr>
</tbody>
</table>

**Examples**

Name the property shown by each statement.

1. \(4 \cdot 11 \cdot 2 = 11 \cdot 4 \cdot 2\) Commutative Property of Multiplication
2. \((n + 12) + 5 = n + (12 + 5)\) Associative Property of Addition

**Your Turn**

a. \((5 \cdot 4) \cdot 3 = 5 \cdot (4 \cdot 3)\)  
b. \(16 + t + 1 = 16 + 1 + t\)

You can use the Commutative and Associative Properties to simplify and evaluate algebraic expressions. To simplify an expression, eliminate all parentheses first and then add, subtract, multiply, or divide.

**Example**

3. Simplify the expression \(15 + (3x + 8)\). Identify the properties used in each step.
   \[
   15 + (3x + 8) = 15 + (8 + 3x) \quad \text{Commutative Property of Addition} \\
   = (15 + 8) + 3x \quad \text{Associative Property of Addition} \\
   = 23 + 3x \quad \text{Substitution Property}
   \]

**Your Turn**

Simplify each expression. Identify the properties used in each step.

a. \(7 + 2a + 6 + 9\)  
b. \((x \cdot 5) \cdot 20\)

d. \((x \cdot 5) \cdot 20\)

**Real World Example**

4. The volume of a box can be found using the expression \(\ell \times w \times h\), where \(\ell\) is the length, \(w\) is the width, and \(h\) is the height. Find the volume of a box whose length is 30 inches, width is 6 inches, and height is 5 inches.
   
   \[
   \ell \times w \times h = 30 \times 6 \times 5 \quad \text{Replace } \ell \text{ with } 30, w \text{ with } 6, \text{ and } h \text{ with } 5. \\
   = 30 \times (6 \times 5) \quad \text{Associative Property of Multiplication} \\
   = 30 \times 30 \quad \text{Substitution Property} \\
   = 900 \quad \text{Substitution Property}
   \]

The volume of the box is 900 cubic inches.
Whole numbers are the numbers 0, 1, 2, 3, 4, and so on. When you add whole numbers, the sum is always a whole number. Likewise, when you multiply whole numbers, the product is a whole number. This is an example of the Closure Property. We say that the whole numbers are closed under addition and multiplication.

Are the whole numbers closed under division? Study these examples.

\[
\begin{align*}
2 \div 1 &= 2 & & \text{ whole number} \\
28 \div 4 &= 7 & & \text{ whole number} \\
5 \div 3 &= \frac{5}{3} & & \text{ fraction}
\end{align*}
\]

It is impossible to list every possible division expression to prove that the Closure Property holds true. However, we can easily show that the statement is false by finding one counterexample. A counterexample is an example that shows the statement is not true. Consider \(5 \div 3\) or \(\frac{5}{3}\). While 5 and 3 are whole numbers, \(\frac{5}{3}\) is not. So, the statement *The whole numbers are closed under division* is false.

**Example 5**

State whether the statement *Division of whole numbers is commutative* is true or false. If false, provide a counterexample.

Write two division expressions using the Commutative Property and check to see whether they are equal.

\[
\begin{align*}
6 \div 3 &= 3 \div 6 & & \text{ Evaluate each expression separately.} \\
2 \neq \frac{1}{2} \\
6 \div 3 &= 2 \text{ and } 3 \div 6 = \frac{1}{2}
\end{align*}
\]

We found a counterexample, so the statement is false. Division of whole numbers is not commutative.

**Your Turn**

e. State whether the statement *Subtraction of whole numbers is associative* is true or false. If false, provide a counterexample.
1. Describe what is meant by the statement The whole numbers are closed under multiplication.

2. Write an equation that illustrates the Commutative Property of Addition.

3. You Try It Abeque says that the expression $(7 \cdot 2) + 5$ equals $7 \cdot (2 + 5)$ because of the Associative Properties of Addition and Multiplication. Jessie disagrees with her. Who is correct? Explain.

4. Name the property shown by each statement. (Examples 1 & 2)

5. Simplify each expression. Identify the properties used in each step. (Example 3)

6. State whether the statement Whole numbers are closed under subtraction is true or false. If false, provide a counterexample. (Example 5)

9. Geology The table shows the number of volcanoes in the United States and Mexico. (Example 4)

   a. Find the total number of volcanoes in these two countries mentally.
   
   b. Describe the properties you used to add the numbers.

<table>
<thead>
<tr>
<th>Location</th>
<th>Number of Volcanoes</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S. Mainland</td>
<td>69</td>
</tr>
<tr>
<td>Alaska</td>
<td>80</td>
</tr>
<tr>
<td>Hawaii</td>
<td>8</td>
</tr>
<tr>
<td>Mexico</td>
<td>31</td>
</tr>
</tbody>
</table>

Source: Kids Discover Volcanoes

Exercises

Practice

10. $(9 \cdot 5) \cdot 20 = 9 \cdot (5 \cdot 20)$

11. $a + 14 = 14 + a$

12. $r \cdot s = s \cdot r$

13. $4 \times 15 \times 25 = 4 \times 25 \times 15$

14. $(7 + 5) + 5 = 7 + (5 + 5)$

15. $c \cdot (d \cdot 10) = (c \cdot d) \cdot 10$

Simplify each expression. Identify the properties used in each step.

16. $h + 1 + 9$

17. $(r \cdot 30) \cdot 5$

18. $17 + k + 23$

19. $6 + (3 + y)$

20. $2 \cdot (19p)$

21. $2 \cdot j \cdot 7$
State whether each statement is **true** or **false**. If false, provide a counterexample.

22. Subtraction of whole numbers is commutative.

23. Division of whole numbers is associative.

24. **Sports** The table shows the point values for different plays in football. The expression below represents the total possible points for a team in a game.

\[
6t + 1x + 3f + 2c + 2s
\]

If a team scores 3 touchdowns, 2 extra points, 2 field goals, and 2 safeties, how many total points are scored?

<table>
<thead>
<tr>
<th>Type of Score</th>
<th>Number of Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>touchdown, ( t )</td>
<td>6</td>
</tr>
<tr>
<td>extra point, ( x )</td>
<td>1</td>
</tr>
<tr>
<td>two-point conversion, ( c )</td>
<td>2</td>
</tr>
<tr>
<td>field goal, ( f )</td>
<td>3</td>
</tr>
<tr>
<td>safety, ( s )</td>
<td>2</td>
</tr>
</tbody>
</table>

25. **Construction** Lumber mills sell wood to lumberyards in *board feet*. The expression shown below represents the number of board feet in a stack of wood.

\[
\text{inches thick} \times \text{inches wide} \times \text{feet long}
\]

Find the number of board feet if the stack of wood is 10 inches thick, 12 inches wide, and 10 feet long.

26. **Geography** The Chattahoochee and Savannah rivers form natural boundaries for the state of Georgia.

   a. Write an expression to approximate the total length of Georgia’s borders using the map at the right.

   b. Evaluate the expression that you wrote in part a. Identify any properties that you used.

27. **Critical Thinking** Use a counterexample to show that subtraction of whole numbers is not associative.

**Mixed Review**

Find the value of each expression. *(Lesson 1–2)*

28. \( 16 \div 2 \cdot 5 \times 3 \)  
29. \( 48 \div [2(3 + 1)] \)  
30. \( 25 - \frac{1}{3}(18 - 9) \)

Evaluate each expression if \( a = 4 \) and \( b = 11 \). *(Lesson 1–2)*

31. \( 196 \div [a(b - a)] \)  
32. \( \frac{ab}{a - 2} \)

**Standardized Test Practice**

33. **Multiple Choice** Which of the following is the value of \( 3t - 5q(r + 1) \), if \( q = 2 \), \( r = 0 \), and \( t = 11 \)? *(Lesson 1–2)*

   A 23  
   B 52  
   C 53  
   D 33
The **Distributive Property** can be applied to simplify expressions. For example, the expression $2 \times (128 + 12)$ can be solved using two different methods.

**Method 1**

$2 \times (128 + 12) = 2(140)$

First, add.

$= 280$

Then, multiply.

**Method 2**

$2 \times (128 + 12) = (2 \times 128) + (2 \times 12)$

First, distribute.

$= 256 + 24$

Multiply.

$= 280$

Add.

The Distributive Property is used in Method 2. Using both methods, the value of the expression is 280.

### What You’ll Learn

You’ll learn to use the Distributive Property to evaluate expressions.

### Why It’s Important

**Shopping** Cashiers use the Distributive Property when they total customers’ groceries. See Exercise 40.

The Distributive Property is used in Method 2. Using both methods, the value of the expression is 280.

### Symbols:

For any numbers $a$, $b$, and $c$,

$ab + ac$ and $ab - ac$.

### Numbers:

$2(5 + 3) = (2 \cdot 5) + (2 \cdot 3)$

$2(5 - 3) = (2 \cdot 5) - (2 \cdot 3)$

In the expression $ab + c$, it does not matter whether $a$ is placed to the left or to the right of the expression in parentheses. So, $(b + c)a = ba + ca$ and $(b - c)a = ba - ca$.

### Examples

**1.** Simplify each expression.

$3(x + 7)$

$3(x + 7) = (3 \cdot x) + (3 \cdot 7)$

$= 3x + 21$  **Distributive Property**

$= 3x + 21$  **Substitution Property**

$5(2n + 8)$

$5(2n + 8) = (5 \cdot 2n) + (5 \cdot 8)$

$= 10n + 40$  **Distributive Property**

$= 10n + 40$  **Substitution Property**

### Your Turn

a. $6(a + b)$

b. $(1 + 3t)9$
A term is a number, variable, or product or quotient of numbers and variables.

### Examples of Terms | Not Terms
--- | ---
7 7 is a number. | 7 + x is the sum of two terms.
t t is a variable. | 8rs + 7y + 6 is the sum of three terms.
5x 5x is a product. | x − y is the difference of two terms.

The numerical part of a term that contains a variable is called the **coefficient**. For example, the coefficient of 2a is 2. **Like terms** are terms that contain the same variables, such as 2a and 5a or 7xy and 3xy.

Consider the expression 5b + 3b + x + 12x.
- There are four terms.
- The like terms are 5b and 3b, x and 12x.
- The coefficients are shown in the table.

<table>
<thead>
<tr>
<th>Term</th>
<th>Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>5b</td>
<td>5</td>
</tr>
<tr>
<td>3b</td>
<td>3</td>
</tr>
<tr>
<td>x</td>
<td>1</td>
</tr>
<tr>
<td>12x</td>
<td>12</td>
</tr>
</tbody>
</table>

The Distributive Property allows us to combine like terms. If \( a(b + c) = ab + ac \), then \( ab + ac = a(b + c) \) by the Symmetric Property of Equality.

\[
2n + 7n = (2 + 7)n \quad \text{Distributive Property} \\
= 9n \quad \text{Substitution Property}
\]

The expressions \( 2n + 7n \) and \( 9n \) are called **equivalent expressions** because their values are the same for any value of \( n \). An algebraic expression is in **simplest form** when it has no like terms and no parentheses.

### Simplify each expression.

3. \( 4x + 9x \)
   \[
   4x + 9x = (4 + 9)x \quad \text{Distributive Property} \\
   = 13x \quad \text{Substitution Property}
   \]

4. \( a + 7b + 3a - 2b \)
   \[
   a + 7b + 3a - 2b = a + 3a + 7b - 2b \\
   = (a + 3a) + (7b - 2b) \\
   = (1 + 3)a + (7 - 2)b \\
   = 4a + 5b \quad \text{Substitution Property}
   \]

### Your Turn

3. \( 5st + 2st \)
4. \( 6 + y + 3z + 4y \)
You can use the Distributive Property to solve problems in different, and possibly simpler, ways.

Write an equation representing the area $A$ of a soccer field given its width $w$ and length $\ell$ as shown in the diagram. Then simplify the expression and find the area if $w$ is 54 yards and $\ell$ is 60 yards.

Method 1

\[
A = w(\ell + \ell) \quad \text{Multiply the total length by the width.}
\]
\[
= 54(60 + 60) \quad \text{Replace } w \text{ with 54 and } \ell \text{ with 60.}
\]
\[
= 54(120) \quad \text{Substitution Property}
\]
\[
= 6480 \quad \text{Substitution Property}
\]

Method 2

\[
A = w\ell + w\ell \quad \text{Add the areas of the smaller rectangles.}
\]
\[
= 54(60) + 54(60) \quad \text{Replace } w \text{ with 54 and } \ell \text{ with 60.}
\]
\[
= 3240 + 3240 \quad \text{Substitution Property}
\]
\[
= 6480 \quad \text{Substitution Property}
\]

Using either method, the area of the soccer field is 6480 square yards.

### Check for Understanding

#### Communicating Mathematics

1. **Write** an algebraic expression with five terms. One term should have a coefficient of three. Also, include two pairs of like terms.

2. **Explain** why $3xy$ is a term but $3x + y$ is not a term.

3. **Determine** which two expressions are equivalent. Explain how you determined your answer.
   - a. $20n + 3p$
   - b. $16n + p - 4n + 2p$
   - c. $16n + 4p + 4n$
   - d. $12p + 3n$
   - e. $12n + 3p$
   - f. $20n - p - 16n + 2p$

#### Guided Practice

**Getting Ready** Name the like terms in each list of terms.

<table>
<thead>
<tr>
<th>Sample: $3c, a, ab, 5, 2c$</th>
<th>Solution: $3c, 2c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4. $5m, 2n, 7n$</td>
<td>5. $8, 8p, 9p, 9q$</td>
</tr>
<tr>
<td>6. $4h, 10gh, 8, 2h$</td>
<td>7. $6b, 6bc, bc$</td>
</tr>
</tbody>
</table>

**Vocabulary**
- term
- coefficient
- like terms
- equivalent expressions
- simplest form
Simplify each expression.  (Examples 1–4)

8. \(5x + 9x\)  
9. \(4y + 2 - 3y\)  
10. \(2(5g + 3g)\)  
11. \(3(4 - 6m)\)  
12. \(8(2s + 7)\)  
13. \((3a + 5t) + (4a + 2t)\)  

14. **School** Every student at Miller High School must wear a uniform. Suppose shirts or blouses cost $18 and skirts or pants cost $25.  
(Example 5)

a. If 250 students buy a uniform consisting of a shirt or blouse and a skirt or pants, write an expression representing the total cost.

b. Find the total cost.

33. Write \(5(2n + 3r) + 4n + 3(r + 2)\) in simplest form. Indicate the property that justifies each step.

34. Is the statement \(2 + (s \cdot t) = (2 + s) \cdot (2 + t)\) true or false? Find values for \(s\) and \(t\) to show that the statement may be true. Otherwise, find a counterexample to show that the statement is false.

35. What is the value of \(6y\) decreased by the quantity \(2y + 1\) if \(y\) is equivalent to 3?

36. What is the sum of \(14xy, xy,\) and \(5xy\) if \(x\) equals 1 and \(y\) equals 4?

37. **Sports** Rich bought two baseballs for $4 each and two basketballs for $22 each. What is the total cost? Use the Distributive Property to solve the problem in two different ways.

38. **Retail** Marie and Mark work at a local department store. Each earns $6.25 per hour. Maria works 24 hours per week, and Mark works 32 hours per week. How much do the two of them earn together each week?

39. **Health** If an adult male’s height is \(h\) inches over five feet, his approximate normal weight is given by the expression \(6.2(20 + h)\).  

a. What should the normal weight of a 5’9” male be?  

b. How many more pounds should the normal weight of a man that is 6’2” tall be than a man that is 5’9” tall?
40. **Shopping** Luanda went to the grocery store and bought the items in the table below.

<table>
<thead>
<tr>
<th>Item</th>
<th>Cost per Item</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>can of soup</td>
<td>$0.99</td>
<td>4</td>
</tr>
<tr>
<td>can of corn</td>
<td>$0.49</td>
<td>4</td>
</tr>
<tr>
<td>bag of apples</td>
<td>$2.29</td>
<td>4</td>
</tr>
<tr>
<td>box of crackers</td>
<td>$3.29</td>
<td>2</td>
</tr>
<tr>
<td>jar of jelly</td>
<td>$2.69</td>
<td>2</td>
</tr>
</tbody>
</table>

a. Use the Distributive Property to write an expression representing the cost of the items.
b. Find the change Luanda will receive if she gives the clerk $30.

41. **Theater** A school’s drama club is creating a stage backdrop with a city theme for a performance. The students sketched a model of buildings as shown at the right.

a. How many square feet of cardboard will they need to make the buildings? Use the expression $\ell w$, where $\ell$ is the length and $w$ is the width of each rectangle, to find the area of each rectangle. Then add to find the total area.

b. Show how to use the Distributive Property as another method in finding the total area of the buildings.

42. **Critical Thinking** Use the Distributive Property to write an expression that is equivalent to $3ax + 6ay$.

43. 8(2 + 6) = (2 + 6)8

44. $(7 + 4) + 3 = 7 + (4 + 3)$

45. If $19 - 3 = 16$, then $16 = 19 - 3$. (Lesson 1–2)

Find the value of each expression. (Lesson 1–2)

46. $8 + 6 ÷ 2 + 2$

47. $3(6 - 32 ÷ 8)$

48. Write an equation for the sentence *Eighteen decreased by d is equal to f*. (Lesson 1–1)

49. **Extended Response** Some toys are 30 decibels louder than jets during takeoff. Suppose jets produce $d$ decibels of noise during takeoff. (Lesson 1–1)

a. Write an expression to represent the number of decibels produced by the loud toys.

b. Evaluate the expression if $d = 140$.

50. **Multiple Choice** Which of the following is an algebraic expression for six times a number decreased by 17? (Lesson 1–1)

   A $6n + 17$  
   B $6n - 17$  
   C $17 + 6n$  
   D $17 - 6n$
In mathematics, solving problems is an important activity. Any problem can be solved using a problem-solving plan like the one below.

1. **Explore**
   Read the problem carefully. Identify the information that is given and determine what you need to find.

2. **Plan**
   Select a strategy for solving the problem. Some strategies are shown at the right. If possible, estimate what you think the answer should be before solving the problem.

3. **Solve**
   Use your strategy to solve the problem. You may have to choose a variable for the unknown, and then write an expression. Be sure to answer the question.

4. **Examine**
   Check your answer. Does it make sense? Is it reasonably close to your estimate?

One important problem-solving strategy is using an equation. An equation that states a rule for the relationship between quantities is called a **formula**.

Money in a bank account earns **interest**. You find simple interest by using the formula $I = prt$.

- $I$ = interest
- $p$ = principal, or amount deposited
- $r$ = interest rate, written as a decimal
- $t$ = time in years

**Example 1**
Suppose you deposit $220 into an account that pays 3% simple interest. How much money would you have in the account after five years?

**Explore**
What do you know?
- The amount of money deposited is $220.
- The interest rate is 3% or 0.03.
- The time is 5 years.
What do you need to find?
• the amount of money, including interest, at the end of five years

Plan
What is the best strategy to use?
Use the formula \( I = \frac{prt}{100} \) and substitute the known values.
Add this amount to the original deposit.

Estimate: 1% of $220 is $2.20. So, 3% of $220 is about \( 3 \times \frac{2}{100} \) or $6 per year. This will be $30 in five years. You should have approximately 220 + 30 or $250 in five years.

Solve

\[ I = \frac{p \cdot r \cdot t}{100} \]

\[ I = 220 \cdot 0.03 \cdot 5 \text{ or } 33 \]

You will earn $33 in interest, so the total amount after five years is $220 + $33 or $253.

Examine
Is your answer close to your estimate?
Yes, $253 is close to $250, so the answer is reasonable.

Science
Use \( F = 1.8C + 32 \) to change degrees Celsius \( C \) to degrees Fahrenheit \( F \). Find the temperature in degrees Fahrenheit if it is 29°C.

Hands-On Algebra

Materials:
- rectangular box
- ruler

Step 1
Label the edges of a rectangular box \( l, w, \) or \( h \) to represent the length, width, and height of the box.

Step 2
Take the box apart so that it lies flat on the table with the labels face up.

Try These
1. Find the area of each rectangular side of the box in terms of the variables \( l, w, \) and \( h \). Be sure to include the top or lid.
2. The sum of the areas is equal to \( S \), the total surface area of a rectangular solid. Express this as a formula in simplest form.
3. Measure the lengths of the sides in centimeters or inches to find the values of \( l, w, \) and \( h \).
4. Use the formula in Exercise 2 to find the total surface area of your box.
How many ways can you make 25¢ using dimes, nickels, and pennies?

**Explore**
A quarter is worth 25¢. How many ways can you make 25¢ without using a quarter?

**Plan**
Make a chart listing every possible combination.

**Solve**

<table>
<thead>
<tr>
<th>Coin</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimes</td>
<td>2 2 1 1 1 1 0 0 0 0 0 0</td>
</tr>
<tr>
<td>Nickels</td>
<td>1 0 3 2 1 0 5 4 3 2 1 0</td>
</tr>
<tr>
<td>Pennies</td>
<td>0 5 0 5 10 15 0 5 10 15 20 25</td>
</tr>
</tbody>
</table>

There are 12 ways to make 25¢.

**Examine**
Check that each combination totals 25¢ and that there are no other possible combinations. The solution checks.

You can use a graphing calculator to solve problems involving formulas.

The area of a trapezoid is $A = \frac{1}{2} h(a + b)$, where $h$ is the height and $a$ and $b$ are the lengths of the bases. Use a graphing calculator to find the area of trapezoid $JKLM$.

$A = \frac{1}{2} \cdot 2.5(3.2 + 6)$

Replace each variable with its value.

Enter: $1 ÷ 2 \times 2.5 (3.2 + 6) \text{ ENTER } \#5$

**Try These**
1. Find the area of trapezoid $JKLM$ if the height is 15 centimeters and the bases remain the same.
2. Find the area of a trapezoid if base $a$ is 14 inches long, base $b$ is 10 inches long, and the height is 7 inches.

The chart below summarizes the properties of numbers. The properties are useful when you are solving problems.

<table>
<thead>
<tr>
<th>Property</th>
<th>Addition</th>
<th>Multiplication</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commutative</td>
<td>$a + b = b + a$</td>
<td>$ab = ba$</td>
</tr>
<tr>
<td>Associative</td>
<td>$(a + b) + c = a + (b + c)$</td>
<td>$(ab)c = a(bc)$</td>
</tr>
<tr>
<td>Identity</td>
<td>$a + 0 = 0 + a = a$</td>
<td>$a \cdot 1 = 1 \cdot a = a$</td>
</tr>
<tr>
<td></td>
<td>0 is the identity.</td>
<td>1 is the identity.</td>
</tr>
<tr>
<td>Zero</td>
<td>$a \cdot 0 = 0 \cdot a = 0$</td>
<td></td>
</tr>
<tr>
<td>Distributive</td>
<td>$ab + c = ab + ac$ and $a(b - c) = ab - ac$</td>
<td></td>
</tr>
<tr>
<td>Substitution</td>
<td>If $a = b$, then $a$ may be substituted for $b$.</td>
<td></td>
</tr>
</tbody>
</table>
1. List three reasons for “looking back” when examining the answer to a problem.

2. Write a problem in which you need to find the surface area of a rectangular solid. Then solve the problem.

For each situation, answer the related questions.

Carlos bought 2 more rock CDs than jazz CDs and 3 fewer country CDs than rock CDs. He bought eight CDs, including 1 classical CD.

3. Did Carlos buy more country than rock?

4. Which type of CD did he buy the most of?

5. If he bought $n$ jazz CDs, how many rock CDs did he buy?

6. Geometry The perimeter $P$ of a rectangle is the sum of two times the length $l$ and two times the width $w$.

   (Example 1)

   a. Write a formula for the perimeter of a rectangle.
   b. What is the perimeter of the rectangle shown above?

7. Money Nate has $267 in bills. None of the bills is greater than $10. He has eleven $10 bills. He has seven fewer $5 bills than $1 bills.

   a. How many $5 and $1 bills does he have?
   b. Describe the problem-solving strategy that you used to solve this problem. (Example 2)

8. Shopping Two cans of vegetables together cost $1.08. One of them costs 10¢ more than the other. (Example 2)

   a. Would 2 cans of the less expensive vegetable cost more or less than $1.08?
   b. How much would it cost to buy 3 cans of each?
Solve each problem. Use any strategy.

9. Craig is 24 years younger than his mother. Together their ages total 56 years. How old is each person? Explain how you found your answer.

10. Moira has $500 in the bank at an annual interest rate of 4%. How much money will she have in her account after two years?

11. Joanne has 20 books on crafts and cooking. She has 6 more cookbooks than craft books. How many of each does she have?

12. How many ways are there to make 20¢ using dimes, nickels, and pennies?

13. Six Explorer Scouts from different packs met for the first time. They all shook hands with each other when they met.
   a. Make a chart or draw a diagram to represent the problem.
   b. How many handshakes were there in all?
   c. The number of handshakes \( h \) can also be found by using the formula
      \[ h = \frac{p(p - 1)}{2}, \]
      where \( p \) represents the number of people.
      How many handshakes would there be among 12 people?
   d. Which strategy would you prefer to use to solve the problem: make a table, draw a diagram, or use a formula?

14. Savings The table shows the cost of leasing a car for 36 months. Which option is a better deal? Explain.

<table>
<thead>
<tr>
<th>Type of Fee</th>
<th>Option A Cost ($)</th>
<th>Option B Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>monthly payment</td>
<td>99</td>
<td>168</td>
</tr>
<tr>
<td>monthly tax</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>bank fee</td>
<td>495</td>
<td>0</td>
</tr>
<tr>
<td>down payment</td>
<td>1956</td>
<td>0</td>
</tr>
<tr>
<td>license plates</td>
<td>75</td>
<td>0</td>
</tr>
</tbody>
</table>

15. Weather Meteorologists can predict when a storm will hit their area by examining the travel time of the storm system. To do this, they use the following formula.

\[
\text{distance from storm (miles)} \div \text{speed of storm (miles per hour)} = \text{travel time of storm (hours)}
\]

At 4:00 P.M. a storm is heading toward the coast at a speed of 30 miles per hour. The storm is about 150 miles from the coast. What time will the storm hit the coast?
16. **Geometry**  The area of a triangle is one-half times the product of the base $b$ and the height $h$.
   a. Write a formula for the area of a triangle.
   b. Find the area of the triangle.

17. **History**  Distance traveled $d$ equals the product of rate $r$ and time $t$.
   a. Write the formula for distance.
   b. In 1936, the *Douglas DC-3* became the first commercial airliner to transport passengers. It flew nonstop from New York to Chicago at an average of 190 miles per hour and the flight lasted approximately 3.7 hours. Find the distance it flew.

18. **Savings**  Refer to the table at the left.
   a. Suppose you saved 50¢ each school day (180 days) while you were in eighth grade. How much would you have to deposit in a bank account at the end of the school year?
   b. Suppose you deposited your “school year savings” in an account with a 4% annual interest rate. How much money would you have in your account after a year?
   c. Suppose you saved the 50¢ each school day (180 days) while you were a freshman in high school. Find the sum of this amount and the money already in your account from the eighth grade.
   d. How much money would you have in your account after the second year?
   e. Repeat steps c and d for your junior and senior years. How much money would you have in your account by the time you graduated from high school? Compare this amount to that listed in the table.

19. **Critical Thinking**  Refer to Exercise 13. In a meeting, there were exactly 190 handshakes. How many people were at the meeting?

---

**Mixed Review**

Simplify each expression.  *Lesson 1–4*

20. $21x - 10x$  
21. $5b + 3b$  
22. $3(x + 2y)$  
23. $9a + 15(a + 3)$

---

**Standardized Test Practice**

24. **Short Response**  *Lesson 1–3*
   State the property shown by $4(ab) = (4a)b$.

25. **Multiple Choice**  *Lesson 1–3*
   The top of a volleyball net is 7 feet 11 inches from the floor. The bottom of the net is 4 feet 8 inches from the floor. How wide is the volleyball net?
   - A 3 feet 3 inches  
   - B 2.31 feet  
   - C 7 feet 1 inch  
   - D 6 feet

---

*Source: Zillions*

*Based on 180 school days in each year * 50 cents a day = $90 a school year.

*www.algconcepts.com/self_check_quiz*
Logical Reasoning

Mathematicians use logical reasoning to discover new ideas and solve problems. *Inductive* and *deductive* reasoning are two forms of reasoning. Let’s investigate them to find out how they differ.

**Investigate**

1. Use the colored paper and the paper punch to make at least 25 dots of each color. You will use these dots to explore patterns.
   a. *Triangular numbers* are represented by the number of dots needed to form different-sized triangles. Use your colored dots to form the first four triangular numbers shown below.

   ![Triangular numbers](image)

   1st number = 1  
   2nd number = 3  
   3rd number = 6  
   4th number = 10

   b. Draw the fifth triangular number. Do you see a pattern? Use the pattern to write the next five triangular numbers.

   c. In Step 1b, you used *inductive reasoning*, where a conclusion is made based on a pattern or past events.

2. *Deductive reasoning* is the process of using facts, rules, definitions, or properties in a logical order. You use deductive reasoning to reach valid conclusions.
   a. Use the following information to reach a valid conclusion.

   Conditional: If I visit the island of Kauai, then I am in Hawaii.

   Given: I visit the island of Kauai.

   The portion of the sentence following *if* is called the hypothesis, and the part following *then* is called the conclusion. This conditional is true since Kauai is a Hawaiian island.
b. On a two-inch square, write the hypothesis “I visit the island of Kauai.” On a three-inch square, write the conclusion “I am in Hawaii.” Place the two-inch square inside the three-inch square.

c. Place your pencil on the given statement, “I visit the island of Kauai.” Since the pencil is also contained within the square with the conclusion, “I am in Hawaii,” the conclusion is valid.

d. Repeat Step 2c, but exchange the hypothesis and conclusion. The new conditional is If I am in Hawaii, then I visit the island of Kauai. You can place your pencil in any region marked by the ×’s in the diagram. You may be in Hawaii, visiting Maui, not the island of Kauai. You cannot reach the conclusion using the conditional and the given information.

In this extension, you will continue to investigate inductive and deductive reasoning.

1. The first four square numbers are 1, 4, 9, and 16. Use the colored dots to make the first five square numbers. Use inductive reasoning to list the first ten square numbers.

2. The first four pentagonal numbers are 1, 5, 12, and 22. Use the colored dots to make the first five pentagonal numbers. Use inductive reasoning to list the first ten pentagonal numbers.

3. For each problem, identify the hypothesis and conclusion. Then use squares as shown above to determine whether a valid conclusion can be made from the conditional and given information.
   a. Conditional: If the living organism is a grizzly bear, then it is a mammal.
      Given: The living organism is a grizzly bear.
   b. Conditional: If Aislyn is in the Sears Tower, then she is in Chicago.
      Given: Aislyn is in Chicago.

4. Write a paragraph explaining the difference between inductive and deductive reasoning. Include an example of each type of reasoning.

Presenting Your Investigation

Here are some ideas to help you present your conclusions to the class.

• Make a poster showing the triangular, square, and pentagonal numbers.
• Include a description of the patterns you observed.

For more information on logical reasoning, visit: www.algconcepts.com
**Sampling** is a convenient way to gather data, or information, so that predictions can be made about a population. A sample is a small group that is used to represent a much larger population. Three important characteristics of a good sample are listed below.

---

**Sampling Criteria**

A good sample is:

- representative of the larger population,
- selected at random, and
- large enough to provide accurate data.

---

A survey can be biased and give false results if these criteria are not followed. Note that there is no given number to make the sample large enough. You must consider each survey individually to see if it is based on a good sample.

---

One hundred people in Lafayette, Colorado, were asked to eat a bowl of oatmeal every day for a month to see whether eating a healthy breakfast daily could help reduce cholesterol. After 30 days, 98 of those in the sample had lower cholesterol. Is this a good sample? Explain.  

**Source:** Quaker Oats

If the people were randomly chosen, then this is a good sample. Also, the sample appears to be large enough to be representative of the population. For example, the results of two or three people would not have been enough to make any conclusions.

---

Determine whether each is a good sample. Explain.

a. Two hundred students at a school basketball game are surveyed to find the students’ favorite sport.

b. Every other person leaving a supermarket is asked to name their favorite soap.

---

After the survey is complete, the gathered data is organized into different types of tables and charts. One way to organize data is by using a **frequency table**. In a frequency table, you use **tally marks** to record and display the frequency of events.
In an experiment, students “charged” balloons by rubbing them with wool. Then the students placed the balloons on a wall and counted the number of seconds they remained. The class results are shown in the chart at the right. Make a frequency table to organize the data.

**Step 1** Make a table with three columns: Time (s), Tally, and Frequency. Add a title.

**Step 2** It is sometimes helpful to use intervals so there are fewer categories. In this case, we are using intervals of size 10.

**Step 3** Use tally marks to record the times in each interval.

**Step 4** Count the tally marks in each row and record this number in the Frequency column.

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>15–24</td>
<td>8</td>
</tr>
<tr>
<td>25–34</td>
<td>9</td>
</tr>
<tr>
<td>35–44</td>
<td>7</td>
</tr>
<tr>
<td>45–54</td>
<td>1</td>
</tr>
</tbody>
</table>

**Your Turn**

c. Make a frequency table to organize the data in the chart at the right.

In Example 2, suppose the science teacher wanted to know how many balloons stayed on the wall no more than 44 seconds. To answer this question, use a cumulative frequency table in which the frequencies are accumulated for each item.

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Frequency</th>
<th>Cumulative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>15–24</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>25–34</td>
<td>9</td>
<td>17</td>
</tr>
<tr>
<td>35–44</td>
<td>7</td>
<td>24</td>
</tr>
<tr>
<td>45–54</td>
<td>1</td>
<td>25</td>
</tr>
</tbody>
</table>

From the cumulative frequency table, we see that 24 balloons stayed on the wall for 44 seconds or less. Or, 24 balloons stayed on the wall for no more than 44 seconds.
Once you have summarized data in a frequency table or in a cumulative frequency table, you can analyze the information and make conclusions.

Owners of a restaurant are looking for a new location. They counted the number of people who passed by the proposed location one afternoon. The frequency table at the right shows the results of their sampling.

A. Which two groups of people passed by the location most frequently?

<table>
<thead>
<tr>
<th>Age of People</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>under 13</td>
<td></td>
<td>7</td>
</tr>
<tr>
<td>teens</td>
<td></td>
<td>10</td>
</tr>
<tr>
<td>20s</td>
<td></td>
<td>18</td>
</tr>
<tr>
<td>30s</td>
<td></td>
<td>42</td>
</tr>
<tr>
<td>40s</td>
<td></td>
<td>36</td>
</tr>
<tr>
<td>50s</td>
<td></td>
<td>19</td>
</tr>
<tr>
<td>60s</td>
<td></td>
<td>11</td>
</tr>
</tbody>
</table>

adults in their 30s and 40s

B. If the restaurant is an ice cream shop aimed at teens during their lunchtimes, is this a good location for the restaurant? Explain.

Since very few teens pass by the location compared to adults, the owners should probably look for another location.

---

**Check for Understanding**

**Communicating Mathematics**

1. **Explain** the difference between a frequency table and a cumulative frequency table.

2. **List** some examples of how a survey might be biased.

**Guided Practice**

**Determine whether each is a good sample.**

**Explain.**

3. Four people out of 500 are randomly chosen at a senior assembly and surveyed to find the percent of seniors who drive to school.

4. Six hundred randomly chosen pea seeds are used to determine whether wrinkled seeds or round seeds are the more common type of seed.
Refer to the chart at the right.

5. Make a frequency table to organize the data. \textit{(Example 2)}

6. What number of goals was scored most frequently? \textit{(Example 3)}

7. How many times did the team score 8 goals? \textit{(Example 3)}

8. How many more times did the soccer team score six goals than three goals? \textit{(Example 3)}

9. \textbf{Technology} When lines of cars get too long at some traffic lights, computers override the signals to turn the lights green and allow the cars to move. A cycle is the number of seconds it takes a light to change from red back to red. The frequency table below shows different traffic light cycles during one afternoon. \textit{(Examples 2 \& 3)}

<table>
<thead>
<tr>
<th>Cycle (s)</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>\textbf{IIII II III III III III III III III III}</td>
<td>33</td>
</tr>
<tr>
<td>90</td>
<td>\textbf{IIII III II III III III III III III III II}</td>
<td>42</td>
</tr>
<tr>
<td>100</td>
<td>\textbf{IIII III II III III III III III III III III III III III III}</td>
<td>60</td>
</tr>
</tbody>
</table>

a. Which cycle occurred the most?

b. Make a cumulative frequency table of the data.

c. If the standard cycle for a traffic light is 100 seconds, how many times during this period was the cycle less than the standard?

\textbf{Exercises}

Determine whether each is a good sample. Describe what caused the bias in each poor sample. Explain.

10. Thirty people standing in a movie line are asked to name their favorite actor.

11. Police stop every fifth car at a sobriety checkpoint.

12. Every other household in a neighborhood of 240 homes is surveyed to determine how many people in the area recycle.

13. Every other household in a neighborhood of 20 homes is surveyed to determine the country’s favorite presidential candidate.

14. Every third student on a class roster is surveyed to determine the average number of hours students in the class spend on a computer.

15. All people leaving a sporting goods store are asked to name their favorite golfer.
16. Refer to the chart at the right.
   a. Make a frequency table to organize the data.
   b. How many fewer sausage pizzas were ordered than cheese pizzas?
   c. Suppose \( x \) mushroom pizzas were also ordered. Write an expression representing the total number of mushroom, vegetable, and pepperoni pizzas ordered. Write the expression in simplest form.

17. Refer to the chart at the right.
   a. Make a frequency table to organize the data.
   b. What was the most common score?
   c. Suppose each \( S \) represents the score and each \( F \) represents the frequency for that score. Explain why the formula below determines the class average \( A \) for this quiz.
      \[
      A = \frac{(S_1 \cdot F_1) + (S_2 \cdot F_2) + \ldots + (S_8 \cdot F_8)}{30}
      \]
   d. Find the class average for the quiz.

18. Refer to the chart at the right.
   a. Make a cumulative frequency table to organize the data.
   b. In how many games were there at least three home runs?
   c. In how many games were there no more than four home runs?

19. Why do you suppose a coffee and bagel shop would want to locate where a lot of people walk past the store between 7:00 A.M. and 10:30 A.M.?

20. Health When you have a blood test taken for your health, why does the technician only take a few vials of your blood? Use the terms you learned in this lesson to explain your answer.

21. Marketing A new cola drink is out on the market. Name three places where the cola company could set up taste tests to determine interest in the drink.
22. **Entertainment**  The frequency table at the right shows students’ favorite types of movies in one class.

   a. Suppose you invite students in this class to a party. What type of movie would you show? Explain.

   b. In another class, three times more students favored drama, and two fewer favored comedy. Write an expression to find the total number of people in that class who favored drama and comedy. Then find the number.

23. **Critical Thinking**  Suppose someone takes a phone survey from a large random sample of people. Do you think that the wording of a question or the surveyor’s tone of voice can affect the responses and cause biased results? Explain.

24. An adult bus ticket and a child’s bus ticket together cost $2.40. The adult fare is twice the child’s fare. What is the adult’s fare? Use any strategy to solve the problem.  
   *Lesson 1–5*

25. **Travel**  What distance can a car travel in 5 hours at a constant rate of 55 miles per hour? Use a diagram or the formula \( d = rt \) to solve the problem.  
   *Lesson 1–5*

26. Simplify the expression \( 16a + 21a + 30b - 7b \).  
   *Lesson 1–4*

27. **Short Response**  Write a verbal expression for \( x + 9 \).  
   *Lesson 1–1*

28. **Short Response**  Write an algebraic expression for \( 4 \times n \) less 3.  
   *Lesson 1–1*

---

**Mixed Review**

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**Standardized Test Practice**

---

**Quiz 2**  **Lessons 1–3 through 1–6**

Simplify each expression. Identify the properties used in each step.  
*Lesson 1–3*

1. \( 11 + 2a + 6 \)

2. \( 4 \cdot (8t) \)

3. **Health**  Your optimum exercise heart rate per minute is given by the expression \( 0.7(220 - a) \), where \( a \) is your age. Use your age for \( a \) and find your optimum exercise heart rate.  
   *Lesson 1–4*

4. **Fitness**  Lorena runs for 30 minutes each day. Find the distance she runs if she averages 660 feet per minute. Use the formula \( d = rt \).  
   *Lesson 1–5*

5. **Biology**  Make a frequency table to organize the data in the chart at the right. Which eye color occurs the least?  
   *Lesson 1–6*

---

**Eye Color**

- H = brown, U = blue, G = green, H = hazel

---

www.algconcepts.com/self_check_quiz
Graphs are a good way to display and analyze data. The graph at the right is a line graph. It shows trends or changes over time. There are no holes in the graph and every point on the graph has meaning. To construct a line graph, include the following items.

1. a title  
2. a label on each axis describing the variable that it represents  
3. equal intervals on each axis

Note that the graph at the right contains all three items.

The number of annual visitors to the Grand Canyon is given in the table at the right. Construct a line graph of the data. Then use the graph to predict the number of annual visitors to the Grand Canyon in the year 2010.

**Step 1** Draw a horizontal axis and a vertical axis and label them as shown below. Include a title.

**Step 2** Plot the points.

**Step 3** Draw a line by connecting the points.

You can see from the graph that the general trend is that the number of visitors to the Grand Canyon increases steadily every ten years. A good prediction for the year 2010 might be about 6 or 6.5 million people.
The table at the right shows the approximate U.S. consumption of bottled water per person. Construct a line graph of the data. Then use it to predict the amount of bottled water each person will drink in the year 2005.

Another type of graph that is used to display data is a histogram. A histogram uses data from a frequency table and displays it over equal intervals. To make a histogram, include the same three items as the line graph: title, axes labels, and equal intervals. In a histogram, all bars should be the same width with no space between them.

The frequency table is from Example 2 in Lesson 1–6. It shows the various time intervals that “charged” balloons remained stuck to the wall. Construct a histogram of the data.

Step 1 Draw a horizontal axis and a vertical axis and label them as shown below. Include the title.

Step 2 Label equal intervals given in the frequency table on the horizontal axis. Label equal intervals of 1 on the vertical axis.

Step 3 For each time interval, draw a bar whose height is given by the frequency.

The histogram gives a better visual display of the data than the frequency table. In Lesson 1–6, we used cumulative frequency tables to organize data. Likewise, we can construct cumulative frequency histograms.
3. The ages of people who participated in a recent survey are shown in the table at the right. Construct a cumulative frequency histogram to display the data.

First, make a cumulative frequency table. Then construct a histogram using the cumulative frequencies for the bar heights. Remember to label the axes and include the title.

### Survey

<table>
<thead>
<tr>
<th>Age</th>
<th>Frequency</th>
<th>Cumulative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–10</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>11–20</td>
<td>9</td>
<td>17</td>
</tr>
<tr>
<td>21–30</td>
<td>5</td>
<td>22</td>
</tr>
<tr>
<td>31–40</td>
<td>12</td>
<td>34</td>
</tr>
<tr>
<td>41–50</td>
<td>8</td>
<td>42</td>
</tr>
</tbody>
</table>

- **Survey Participants**

- **Your Turn**

b. Construct a cumulative frequency histogram of the data in Example 2.

Another way to display data is a **stem-and-leaf plot**.

- **Stem | Leaf**
  - The greatest common place value for each data item is used to form the stem.
  - The leaves are formed by the next greatest place value. The leaves are formed by the next greatest place value.
  - A key is always included. This shows how the digits are related.
  - In this case, the tens digits are the stems. The ones digits are the leaves. Write the leaves in order from least to greatest.

### Stem-and-Leaf Plot Example

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 6</td>
</tr>
<tr>
<td>2</td>
<td>1 3 9</td>
</tr>
<tr>
<td>3</td>
<td>5 5</td>
</tr>
<tr>
<td>4</td>
<td>2 3</td>
</tr>
</tbody>
</table>

In the stem-and-leaf plot at the right, the data are represented by three-digit numbers. In this case, use the digits in the first two place values to form the stems. For example, the values for 102, 108, 114, 115, 125, 127, 131, and 139 are shown in the stem-and-leaf plot at the right.

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>2 8</td>
</tr>
<tr>
<td>11</td>
<td>4 5</td>
</tr>
<tr>
<td>12</td>
<td>5 7</td>
</tr>
</tbody>
</table>
| 13   | 1 9  | 11 5 = 115
The table shows the class results on a 50-question test. Make a stem-and-leaf plot of the grades.

The tens digits are the stems, so the stems are 1, 2, 3, and 4. The ones digits are the leaves.

Now arrange the leaves in numerical order to make the results easier to observe and analyze.

What were the highest and lowest scores?
49 and 13

Which score occurred most frequently?
29 and 37, three times each

How many students received a score of 35 or better?
11 students

c. Make a stem-and-leaf plot of the quiz grades below.
54, 55, 60, 42, 41, 75, 50, 68, 62, 54, 70, 50

Check for Understanding

1. Explain the differences between the use of line graphs and histograms.

2. Identify each essential part of a correctly drawn line graph or histogram.

3. Marcia says that a histogram works as well as a line graph to show trends over time. Manuel says that a histogram shows intervals, not trends. Who is correct? Explain.
Guided Practice

The table at the right shows the percent of homes in California with internet access. (Example 1)

4. Make a line graph of the data.

5. Between which two years was the growth of on-line access the greatest?

6. Predict the percent of homes with on-line access in the year 2001.

Refer to the histogram at the right.

7. Determine the length of most maple leaves. (Example 2)

8. How many leaves were sampled? (Example 2)

9. Construct a cumulative frequency histogram of the data. (Example 3)

10. How many of the leaves were no more than 15 centimeters long? (Example 3)

11. Weather The stem-and-leaf plot at the right shows the daily high temperatures in McComb, Mississippi, in March. (Example 4) Source: The Weather Underground

   a. What was the highest temperature?

   b. On how many days was the high temperature in the 70s?

   c. What temperature occurred most frequently?

Exercises

Practice

The percent of unemployment among workers ages 16 to 19 is shown at the right.

12. Make a line graph of the data.

13. When was unemployment at its highest?

14. Describe the general trend in unemployment among teens ages 16 to 19.
In a survey, men and women were asked how long they were willing to stay on hold when calling a customer service representative about a product they purchased. The results are shown in the table at the right.

15. Make a histogram showing the men’s responses.
16. Make a histogram showing the women’s responses.
17. How do your histograms compare?
18. Who do you think would hang up the phone sooner, men or women?

The stem-and-leaf plot at the right gives the number of catches of the NFL’s leading pass receiver for the first 39 seasons.

19. What was the greatest number of catches during a season?
20. How many seasons are represented?
21. What number of catches occurred most frequently?
22. How many leading pass receivers have at least 90 catches?

23. **Critical Thinking**  Back-to-back stem-and-leaf plots are used to compare two sets of data. The back-to-back stem-and-leaf plot below compares the performance of two algebra classes on their first test. Which class do you believe did better on the test? Why do you think so?

<table>
<thead>
<tr>
<th>First Period</th>
<th>Stem</th>
<th>Second Period</th>
<th>Stem</th>
</tr>
</thead>
<tbody>
<tr>
<td>9 8</td>
<td>5</td>
<td>7 8</td>
<td>6</td>
</tr>
<tr>
<td>9 8 7</td>
<td>6</td>
<td>2 4 4 4 5 5 8</td>
<td>7 2</td>
</tr>
<tr>
<td>7 7 6 5 3</td>
<td>7</td>
<td>1 3 4 5 7</td>
<td>8</td>
</tr>
<tr>
<td>5 4 1</td>
<td>8</td>
<td>0 1 1 3 8</td>
<td>9</td>
</tr>
</tbody>
</table>

Determine whether each is a good sample.  

24. A survey is taken in Alaska to determine how much money an average family in the United States spends on heating their home.
25. In a survey, every third name in the phone book is called and the person answering is interviewed.

26. Write a formula for the perimeter P of a square with side s in simplest form.  

27. **Short Response**  Write 5x + 3(x − y) in simplest form.  
28. **Multiple Choice**  Evaluate 12 · 6 + 3 · 2 − 8.  

A 142  B −144  C 70  D 120
Understanding and Using the Vocabulary

After completing this chapter, you should be able to define each term, property, or phrase and give an example of each.

**Algebra**
- algebraic expression (p. 4)
- coefficient (p. 20)
- equation (p. 5)
- equivalent expressions (p. 20)
- evaluating (p. 10)
- factors (p. 4)
- formula (p. 24)
- like terms (p. 20)
- numerical expression (p. 4)
- order of operations (p. 8)
- product (p. 4)
- quotient (p. 4)
- simplest form (p. 20)

**Statistics**
- cumulative frequency
- histogram (p. 39)
- cumulative frequency table (p. 33)
- data (p. 32)
- frequency table (p. 33)
- histogram (p. 39)
- line graph (p. 38)

Choose the correct term to complete each sentence.

1. A (coefficient, term) is a number, a variable, or a product or quotient of numbers and variables.
2. The result of two numbers multiplied together is the (factor, product).
3. A(n) (numerical expression, algebraic expression) contains variables.
4. According to the (order of operations, like terms), you do multiplication before addition.
5. A (counterexample, hypothesis) shows that a statement is not always true.
6. Some examples of (like terms, whole numbers) are 2x, 10x, and –6x.
7. A (sample, variable) is a group used to represent a much larger population.
8. Any sentence that contains an equals sign is a(n) (equation, formula).
9. Using (sampling, frequency tables) is a way to organize data.
10. A (histogram, stem-and-leaf plot) makes it easier to identify specific data items.

**Skills and Concepts**

**Objectives and Examples**

- **Lesson 1–1** Translate words into algebraic expressions and equations.

Write an algebraic expression for the verbal expression *7 decreased by the quantity x divided by 2*.

\[ 7 - \left( \frac{x}{2} \right) \text{ or } 7 - \frac{x}{2} \]

**Review Exercises**

**Write an algebraic expression.**

11. the product of 5 and \( n \)
12. the sum of 2 and three times \( x \)

**Write an equation for each sentence.**

13. Six less than two times \( y \) equals 14.
14. The quotient of 20 and \( x \) is 4.

www.algconcepts.com/vocabulary_review
Chapter 1 Study Guide and Assessment

Objectives and Examples

• **Lesson 1–2** Use the order of operations to evaluate expressions.

\[ 2 \cdot 7 + 2 \cdot 3 = 14 + 6 = 20 \]

• **Lesson 1–3** Use the commutative and associative properties to simplify expressions.

Name the property shown by \(3 + x + 2 = 3 + 2 + x\). Then simplify.

\[ 3 + x + 2 = 3 + 2 + x \quad \text{Commutative (+)} \]

\[ = 5 + x \quad \text{Substitution} \]

• **Lesson 1–4** Use the Distributive Property to evaluate expressions.

\[ 5b + 3(b + 2) = 5b + 3 \cdot b + 3 \cdot 2 \]
\[ = 5b + 3b + 6 \]
\[ = (5 + 3)b + 6 \]
\[ = 8b + 6 \]

• **Lesson 1–5** Use the four-step plan to solve problems.

**Explore**

What do you know? What are you trying to find?

**Plan**

How will you go about solving this? What problem-solving strategy could you use?

**Solve**

Carry out your plan. Does it work? Do you need another plan? If necessary, choose a variable for an unknown and write an expression.

**Examine**

Check your answer. Does it make sense? Is it reasonably close to your estimate?

Review Exercises

Find the value of each expression.

15. \(3 + 8 \div 2\)  
16. \(12 \div 4 + 15 \cdot 3\)  
17. \(29 - 3(9 - 4)\)  
18. \(4(11 + 7) - 9 \cdot 8\)  
19. Find the value of \(3ac - b\) if \(a = 6, b = 9,\) and \(c = 1.\)

Name the property shown by each statement. Then simplify.

20. \(6 + (7 + b) = (6 + 7) + b\)  
21. \(2 \cdot c \cdot 10 = 2 \cdot 10 \cdot c\)  
22. \(9 \cdot (5 \cdot f) = (9 \cdot 5) \cdot f\)  
23. \(x(5 + 4) = (5 + 4)x\)  
24. \(3 + a + 8 = 3 + 8 + a\)  
25. \((g + 1) + 2 = g + (1 + 2)\)

Simplify each expression.

26. \(4(8 + y)\)  
27. \(7(v - 1)\)  
28. \(10x + x\)  
29. \(h(2 + a)\)  
30. \(5z + 2z - 6\)  
31. \(10 + 3(4 - d)\)

Use the four-step plan to solve each problem.

32. **Finance** Mr. Rockwell deposited $1000 in an account that pays 2% interest. How much money would he have in the account after ten years?

33. **School** Jamal is typing a three-page report with approximately 400 words per page for school. He thought he could finish typing the report in 2 hours. After \(1 \frac{1}{2}\) hours, he had finished 2 pages.

a. How many words are in his paper?

b. About how many words had Jamal typed in \(1 \frac{1}{2}\) hours?
Chapter 1 Study Guide and Assessment

Objectives and Examples

- **Lesson 1–6** Collect and organize data using sampling and frequency tables.

  Make a frequency table for the data \(\{1, 4, 3, 4, 0, 2, 3, 1, 0, 2, 0, 4, 0, 0, 4, 1, 2\}\).

<table>
<thead>
<tr>
<th>Number</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>III</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>I</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>II</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>II</td>
<td>4</td>
</tr>
</tbody>
</table>

- **Lesson 1–7** Construct and interpret line graphs, histograms, and stem-and-leaf plots.

  Construct a histogram for the data \(\{10, 10, 10, 10, 11, 11, 12, 13, 14, 14, 14, 15, 15\}\).

  Use the histogram at the left to answer each question.

  40. How large is each interval?
  41. Which interval has the most data?
  42. How many numbers have a value greater than 11?
  43. Make a cumulative histogram from the data.
  44. How many numbers are in the sample?
  45. How many numbers have a value less than 14?

<table>
<thead>
<tr>
<th>Interval</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>10–11</td>
<td>3</td>
</tr>
<tr>
<td>12–13</td>
<td>5</td>
</tr>
<tr>
<td>14–15</td>
<td>2</td>
</tr>
</tbody>
</table>

  Applications and Problem Solving

  46. **Geometry** Write an equation to represent the perimeter \(P\) of the figure below. Then solve for \(P\) if \(x = 9\) and \(y = 5\). *(Lesson 1–5)*

  ![Geometry Figure]

  47. **Testing** The stem-and-leaf plot below shows the scores from a driver’s test. *(Lesson 1–7)*

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>2 8</td>
</tr>
<tr>
<td>7</td>
<td>4 5 5 6</td>
</tr>
<tr>
<td>8</td>
<td>0 4 8 8</td>
</tr>
<tr>
<td>9</td>
<td>2 4 7</td>
</tr>
<tr>
<td>10</td>
<td>0 7</td>
</tr>
</tbody>
</table>

  a. What were the highest and lowest scores?
  b. Which score occurred most frequently?
  c. How many people received a score of 76 or better?
1. **Explain** why we use the order of operations in mathematics.

2. **List** three like terms with the variable \( k \).

**Write an algebraic expression for each verbal expression.**

3. \( x \) increased by 12

4. The quotient of 5 and \( y \)

5. 1 less than 8 times \( p \)

**Use the order of operations to find the value of each expression.**

6. \( \frac{13}{4} \div \frac{1}{5} \)

7. \( 12 + 6 \div 3 - 4 \)

8. \( 3(8 + 2) - 7 \)

**Evaluate each expression if \( h = 8, j = 3, \) and \( k = 2 \).**

9. \( k(4 + j) + 6 \)

10. \( \frac{h + k}{h - j} \)

**Name the property shown by each statement.**

11. If \( 11 = 7 + x \), then \( 7 + x = 11 \).

12. \( 28 \cdot 1 = 28 \)

13. \((r \cdot 9) \cdot 3 = r \cdot (9 \cdot 3)\)

14. \(10 + b = b + 10\)

15. \(6(m + 2) = 6 \cdot m + 6 \cdot 2\)

**Simplify each expression.**

16. \( n + 5n \)

17. \(6x - 4x + 9y - 4y\)

18. \(4(2s + 8t - 1)\)

19. **Sports** Danny stayed late after every basketball practice to shoot 5 free throws. The chart shows how many free throws he made out of 5 for each night of practice.

   a. Make a frequency table to organize the data.

   b. If Danny has basketball practice 5 days a week, how many weeks did he stay late, shooting free throws?

   c. What number of free throws did he make most often?

   d. How many times did he not make any free throws?

   e. How many times did he make all 5 free throws?

20. **Communication** The line graph shows the growth in sales of prepaid calling cards.

   a. Between which two years was growth in sales the greatest?

   b. Predict the number of sales for the year 2002.
Number Concept Problems

Standardized tests include many questions written with realistic settings. Read each question carefully. Be sure you understand the situation and what the question asks.

A calculator can help, but you can often find the answer faster with a pencil and your own math skills. Since standardized tests are timed, you will want to find the correct answers as quickly as possible.

State Test Example

Mrs. Lopez estimates that \( \frac{2}{3} \) of the families in her neighborhood will participate in the annual garage sale. If there are 225 families in her neighborhood, how many families does she expect to participate?

A 75  B 150  C 175  D 220

Hint Estimate the answer before making any calculations.

Solution First estimate. Since \( \frac{2}{3} \) is greater than \( \frac{1}{2} \), more than one half of the 225 families will participate. One half of 225 is about 112. So, choice A is not possible.

The word \( \frac{2}{3} \) (of the families) tells you to use multiplication.

\[
\frac{2}{3} \cdot 225 = \frac{2(225)}{3} \quad \text{Multiply.}
\]

\[
= \frac{2(225)}{3} = \frac{450}{3} = 150
\]

If you use your calculator, multiply 2 by 225, and then divide the answer by 3.

Two-thirds of the 225 families is 150 families. So, the answer is B.

SAT Example

Jan drove 144 miles between 10:00 A.M. and 12:40 P.M. What was her average speed in miles per hour?

Hint Pay attention to the units of measure.

Solution From 10:00 A.M. to 12:40 P.M. is 2 hours and 40 minutes. You need time in hours. Convert minutes to hours.

\[
2 \text{ hours } 40 \text{ minutes } = 2 \frac{40}{60} \text{ hours } = 2 \frac{2}{3} \text{ hours}
\]

\[
\frac{144}{2\frac{2}{3}} = \frac{144}{\frac{8}{3}} = 144 \cdot \frac{3}{8} = \frac{144 \cdot 3}{8} = \frac{18 \cdot 3}{1}
\]

Divide the miles by time.

Rename \( 2\frac{2}{3} \) as \( \frac{8}{3} \).

Multiply by the reciprocal of \( \frac{8}{3} \).

Divide by the GCF, 8.

The answer is 54 miles per hour. Record it on the grid.

- Start with the left column.
- Write the answer in the boxes at the top. Write one digit in each column.
- Mark the corresponding oval in each column.
- Never grid a mixed number; change it to a fraction or a decimal.
After you work each problem, record your answer on the answer sheet provided or on a sheet of paper.

**Multiple Choice**

1. Grant purchased a shirt for $29.95 and 2 pairs of socks for $2.95 a pair. The sales tax on these purchases was $2.42. What was the total amount Grant spent?
   A $35.32  
   B $35.85  
   C $38.27  
   D $39.37

2. Ariel adds $\frac{1}{2}$ cup of flour to a bowl that already has $3\frac{2}{3}$ cups of flour. How many total cups of flour will be in the bowl?
   A $7\frac{1}{3}$  
   B $4\frac{1}{6}$  
   C 4  
   D $3\frac{1}{6}$

3. The number 1134 is divisible by all of the following except—
   A 3  
   B 6  
   C 9  
   D 12  
   E 14.

4. Dr. Hewson has 758 milliliters of a solution to use for a class lab experiment. She divides the solution evenly among 32 students. If 22 milliliters are left after the experiment, how much of the solution did she give each student?
   A 23.0 mL  
   B 24.2 mL  
   C 33.0 mL  
   D 35.9 mL

5. For shipping and handling, a company charges $2.75 in addition to $1.25 for each $10 ordered. Which equation represents the cost $c$ for shipping an order worth $50?
   A $\frac{c}{50} = 2.75 + 1.25$  
   B $c + 2.75 = 1.25 + \frac{50}{10}$  
   C $c = 2.75 + 1.25\left(\frac{50}{10}\right)$  
   D $c = \frac{50}{10}(2.75) + 1.25$

6. Baseballs are packed one dozen per box. There are 208 baseballs to be packed. How many more baseballs will be needed to fill the last, partially filled box?
   A 0  
   B 4  
   C 8  
   D 12  
   E 18

7. Franco is making a casserole. The recipe uses 8 cups of macaroni and serves 12 people. How many cups of macaroni does the recipe use per person?
   A $\frac{2}{3}$  
   B 1  
   C $\frac{1}{2}$  
   D $\frac{1}{3}$

8. Use the commutative and associative properties to compute the product.
   $2 \cdot 4 \cdot 2.5 \cdot 15 \cdot 5 \cdot 10$
   A 1500  
   B 12,000  
   C 15,000  
   D 120,000

**Grid In**

9. The daily newspaper always follows a particular format. Each even-numbered page contains 6 articles, and each odd-numbered page contains 7 articles. If today’s paper has 36 pages, how many articles does it contain?

**Extended Response**

10. The average annual snowfall in Denver, Colorado, is 59.8 inches. How many feet of snow can Denver residents expect in the next 4 years?

   Part A  List the operations you use to solve this problem. Calculate the answer to the nearest hundredth of a foot. Show your work.

   Part B  Round your answer to the nearest foot.